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for
Digital Communication

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To our parents and families
who love and support us
and
to our teachers and students
who enriched our knowledge

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Preface

This book has been designed as a reference book for students or engineers studying communication systems possibly in the curriculum of Electrical Engineering program rather than a text book for any course on communication. Readers are supposed to have taken at least two junior-level courses, one on signals and systems and another one on probability and random processes. In other words, readers should have a basic knowledge about the linear system, Fourier transform, Laplace transform, z-transform, probability, and random processes although the first two chapters of this book provide a brief overview of some background topics to minimize the necessity of the prerequisite courses and to refresh their memory if nothing else.

It is not the aim of this book to provide any foundation in the basic theory of digital communication since the authors do not have such a deep knowledge as to do it. The first aim of this book is to help the readers understand the concepts, techniques, terminologies, equations, and block diagrams appearing in the existing books on communication systems while using MATLAB® to simulate the various communication systems most of which are described by block diagrams and equations. Needless to say, the readers are recommended to learn some basic usage of MATLAB® that is available from the MATLAB help function or the on-line documents at the web site <http://www.mathworks.com/matlabcentral/>. However, they are not required to be so good at MATLAB® since most programs in this book have been composed carefully and completely so that they can be understood in connection with related/referred equations and/or block diagrams. The readers are expected to get used to MATLAB software while trying to modify/use the MATLAB® codes and Simulink® models in this book for solving the end-of-chapter problems or their own problems. The second and main aim of this book is to make even a novice at both MATLAB® and communication systems become acquainted, at least comfortable, with MATLAB® as well as communication systems while running the MATLAB programs on his/her computer and trying to understand what is going on in the systems simulated by the programs. Is it too much to expect that a novice will become interested in communications and simultaneously fall in love with MATLAB®, which is a universal language for engineers and scientists after having read this book through? Is it just the authors' imagination that the readers would think of this book describing and explaining many concepts in MATLAB® rather than in English? In any case, the authors have no intention to hide their hope that this book will be one of the all-the-time-reserved books in most libraries and can be found always on the desks of most communication engineers. The features of this book can be summarized as follows:

1. This book presents more MATLAB programs for the simulation of communication systems than any existent books with the same or similar titles as an approach to explain most things using MATLAB® and figures rather than English and equations.
2. Most MATLAB programs are presented in a complete form so that the readers can run them instantly with no programming skill and focus on understanding the behavior and characteristic of the simulated systems and making interpretations based on the tentative and final simulation results.
3. Many programs have a style of on-line processing rather than batch processing so that the readers can easily understand the whole system and the underlying algorithm in details block by block and operation by operation. Furthermore, the on-line processing style of the programs is expected to let the readers develop their insight into the real system.
4. Authors never think that this book can replace the existent books made by many great authors to whom they are not comparable to. They neither expect that this book can take the place of the MATLAB manual. Instead, this book is designed to play a role of bridge

between MATLAB® software and the theory, block diagrams, and equations appearing in the field of communications so that the readers can feel free to utilize MATLAB® software for studying communication systems and become much more interested in communications than before reading this book.

The contents of this book are derived from the works of many (known or unknown) great scientists, scholars, and researchers, all of whom are deeply appreciated. We would like to thank the reviewers for their valuable comments and suggestions, which contribute to enriching this book.

We also thank the people of the School of Electronic & Electrical Engineering, Chung-Ang University for giving us an academic environment. Without affections and supports of our families and friends, this book could not be written. Special thanks should be given to Senior Researcher Yong-Suk Park for his invaluable help in correction. We gratefully acknowledge the editorial and production staff of A-Jin Publishing Company for their kind, efficient, and encouraging guide.

Program files can be downloaded from <http://wyyang53.com.ne.kr/>. Any questions, comments, and suggestions regarding this book are welcome and they should be mailed to wyyang53@hanmail.net.

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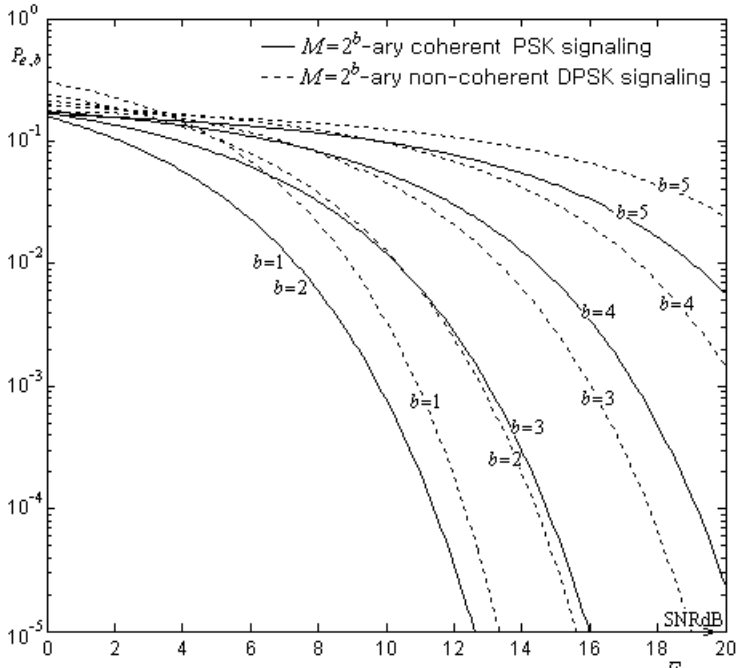


Figure 7.10 The bit error probability vs. $SNRdB_{r,b}=10\log_{10}\frac{E_b}{N_0/2}$ with $M=2^b$-ary PSK/DPSK signalings

7.5 QUADRATURE AMPLITUDE MODULATION (QAM)

The passband $M=2^b$-ary QAM signaling uses the waveforms which have different amplitudes and phases depending on what data they are carrying and therefore, it can be viewed as a kind of APK (amplitude-phase keying), which combines amplitude modulation and phase modulation. Each of the passband $M=2^b$-ary QAM signal waveforms can be written as

$$\begin{aligned} s_m(t) &= A_{mc}s_{uc}(t) + A_{ms}s_{us}(t) = \mathrm{Re}\left\{(A_{mc}+jA_{ms})\sqrt{\frac{2}{T_s}}e^{j\omega_c t}\right\} \quad \text{for } m=0,\,1,\cdots,M-1 \\ &= A_{mc}\sqrt{\frac{2}{T_s}}\cos(\omega_c t) - A_{ms}\sqrt{\frac{2}{T_s}}\sin(\omega_c t) \quad \text{for } 0\le t<T_s \\ &= A_m\sqrt{\frac{2}{T_s}}\cos(\omega_c t+\theta_m) \ \text{ with } A_m=\sqrt{A_{mc}^2+A_{ms}^2} \text{ and } \theta_m=\tan^{-1}\frac{A_{ms}}{A_{mc}} \end{aligned} \tag{7.5.1}$$

where

$$s_{uc}(t)=\sqrt{\frac{2}{T_s}}\cos(\omega_c t),\quad s_{us}(t)=-\sqrt{\frac{2}{T_s}}\sin(\omega_c t):\ \text{Basis signal waveforms} \tag{7.5.2}$$

T_b : Bit time or bit duration, $T_s=bT_b$: Symbol time or symbol duration

$E_s=bE_b$: Signal energy per symbol

The QAM signal waveforms are illustrated in Fig. 7.1(a4) and each of them can be represented as a vector of length A_m

$$\mathbf{s}_m = \begin{bmatrix} A_{mc} & A_{ms} \end{bmatrix} = A_m \begin{bmatrix} \cos\theta_m & \sin\theta_m \end{bmatrix} \tag{7.5.3}$$

and depicted in the signal space as Fig. 7.1(b4) or Fig. 7.11 where the (orthonormal) bases of the signal space are the unit vectors representing $s_{uc}(t)$ and $s_{us}(t)$ defined by Eq. (7.5.2).

Suppose the amplitudes/phases of the $M = 2^b$-ary QAM signal waveforms are designed in such a way that they can be represented by a rectangular constellation in the signal space as Fig. 7.11(a) and the minimum distance among the signal points is $2A$. Then, the average $E_{s,av}$ of signal powers (represented by the squared distance between signal points and the origin) and the average number N_b of adjacent signal points for a signal point vary with the modulation order $M = 2^b$ or the number b of bits per symbol as

$$M = 2^2 = 4: \quad E_{s,av} = \frac{1}{M}\sum_{m=1}^{M-1} A_m^2 = \frac{4A^2 \times (1^2) \times 2}{4} = 2A^2 = \frac{2(M-1)}{3}A^2, \quad N_b = 2 = 4 - \frac{4}{\sqrt{M}=2}$$

$$M = 2^4 = 16: \quad E_{s,av} = \frac{1}{M}\sum_{m=1}^{M-1} A_m^2 = \frac{4A^2 \times (1^2+3^2) \times 4}{16} = 10A^2 = \frac{2(M-1)}{3}A^2$$

$$N_b = \frac{4\times((2-1)^2\times 4+(4-2)\times 3+2)}{4^2} = 3 = 4 - \frac{4}{\sqrt{M}=4}$$

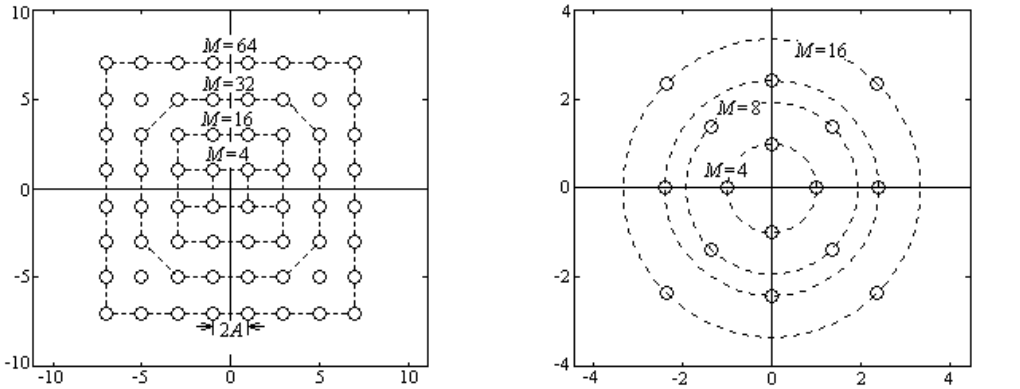
$$M = 4\times 2^4 = 64: \quad E_{s,av} = \frac{4A^2 \times (1^2+3^2+5^2+7^2) \times 8}{M=64} = 42A^2 = \frac{2(M-1)}{3}A^2$$

$$N_b = \frac{4\times((4-1)^2\times 4+(8-2)\times 3+2)}{8^2} = \frac{7}{2} = 4 - \frac{4}{\sqrt{M}=8}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$E_{s,av} = \frac{2(M-1)}{3}A^2 : \text{Average signal energy per symbol} \tag{7.5.4a}$$

$$N_b = 4 - \frac{4}{\sqrt{M}} : \text{Average number of adjacent signal points} \tag{7.5.4b}$$



(a) A rectangular QAM signal constellation diagram (b) A circular QAM signal constellation diagram

Figure 7.11 QAM signal constellation diagrams

A half of the minimum distance $d_{\min}$ among the $M=2^b$-ary QAM signal points in the signal space can be expressed in terms of the average signal energy $E_{b,av}$ per bit as

$$\frac{d_{\min}}{2} = A \overset{(7.5.4a)}{=} \sqrt{\frac{3/2}{M-1}E_{s,av}} = \sqrt{\frac{3/2}{M-1}bE_{b,av}} \tag{7.5.5}$$

Note that if we use the circular constellation as in Fig. 7.11(b), we may have a larger minimum distance with the same average energy $E_{b,av}$, but the difference is very small for $M \geq 16$. Besides, an $M=2^b(b=2m:\text{an even number})$-ary QAM signaling with rectangular signal constellation can easily be implemented by two independent $2^{b/2}$-ary PAM signaling, each of which uses one of the quadrature carriers $\cos(\omega_c t)$ and $\sin(\omega_c t)$, respectively (see Fig. 7.12). This is why QAM signaling with rectangular signal constellation is widely used.

For an $M=2^b=LN$-ary QAM signaling implemented by combining an L-ary PAM signaling and an N-ary PAM signaling, the symbol error probability can be found as

$$\begin{aligned} P_{e,s}(M=LN) &= 1-P(\text{probability of correct detection}) = 1-(1-P_{e,s}(L))(1-P_{e,s}(N)) \\ &\overset{(7.1.5)}{=} 1-\left[1-\frac{2(L-1)}{L}Q\left(\sqrt{\frac{3b/2}{L^2-1}SNR_{r,b}}\right)\right]\left[1-\frac{2(N-1)}{N}Q\left(\sqrt{\frac{3b/2}{N^2-1}SNR_{r,b}}\right)\right] \qquad (7.5.6) \\ &\leq \frac{4(L-1)}{L}Q\left(\sqrt{\frac{3b/2}{M-1}SNR_{r,b}}\right) \text{ with } L \geq N \qquad (7.5.7) \end{aligned}$$

where the upperbound on the RHS coincides with what is obtained by substituting Eqs. (7.5.4b) and (7.5.5) into Eq. (5.2.41). How about the bit error probability? Under the assumption that the information symbols are Gray-coded so that the codes for adjacent signal points differ in only one bit, the most frequent symbol errors contain just one of the b bits mistaken and the relationship between the symbol and bit errors can be written as

$$P_{e,b} = \frac{1}{b}P_{e,s} \tag{7.5.8}$$

Now, let us think about the structure of the $M=2^b=2^{2m}(b=2m:\text{an even number})$-ary QAM communication system depicted in Fig. 7.12 where the XMTR divides the b bits of a message symbol data into two parts of $m=b/2$ bits, converts them to analog signals, and modulates them with the *quadrature carrier*s that are the basis signal waveforms

$$s_{uc}(t) = \sqrt{\frac{2}{T_s}}\cos(\omega_c t) \quad \text{and} \quad s_{us}(t) = -\sqrt{\frac{2}{T_s}}\sin(\omega_c t), \tag{7.5.10}$$

respectively. This QAM scheme is basically equivalent to performing two independent quadrature PAMs in parallel. The RCVR has two *quadrature correlator*s, each of which computes a correlation of the received signal $r(t)$ with $s_{uc}(t)$ and $s_{us}(t)$ to make the sampled outputs

$$y_{c,k} \overset{(5.1.27)}{=} \int_0^{T_s} s_{uc}(t)r(t+(k-1)T_s)\,dt \quad \text{and} \quad y_{s,k} \overset{(5.1.27)}{=} \int_0^{T_s} s_{us}(t)r(t+(k-1)T_s)\,dt, \tag{7.5.11}$$

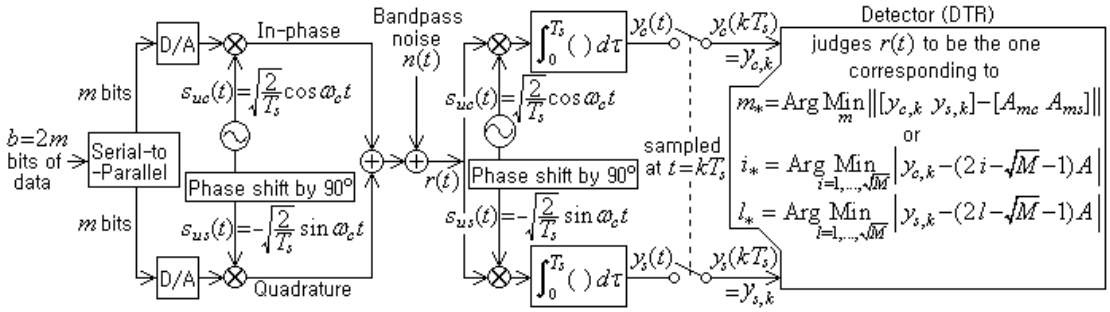


Figure 7.12 The structure of an $M=2^b=2^{2m}=L^2$-ary QAM communication system

respectively. The DTR judges the received signal to be the one represented by the signal point which is the closest to the point $(y_{c,k}, y_{s,k})$ in the signal space as

$$m_* = \text{Arg}\,\underset{m}{\text{Min}}\,\|[y_{c,k}\;\; y_{s,k}]-[A_{mc}\;\; A_{ms}]\| \;=\; \text{Arg}\,\underset{m}{\text{Min}}\,\{(y_{c,k}-A_{mc})^2+(y_{s,k}-A_{ms})^2\} \qquad (7.5.12)$$

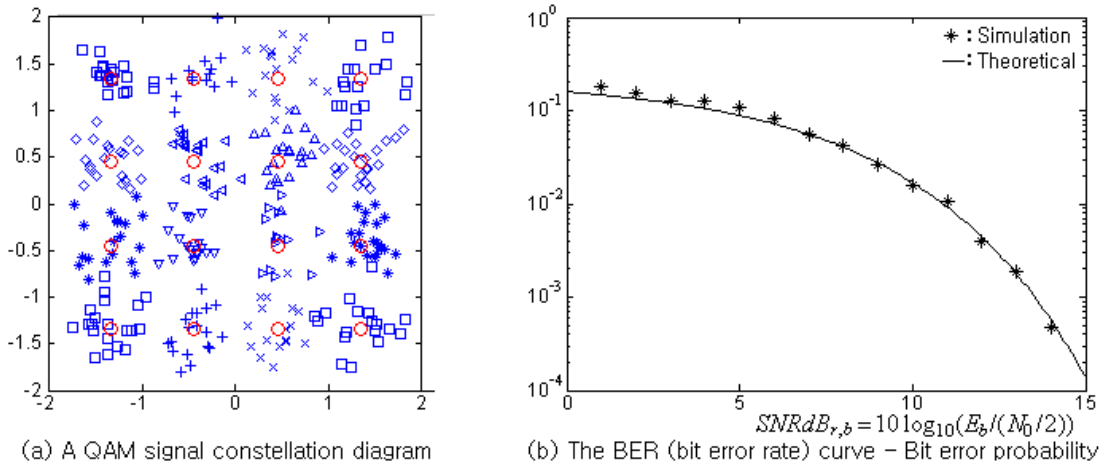
or combines two independent quadrature PAM demodulation results

$$i_* = \text{Arg}\,\underset{i=1,\ldots,\sqrt{M}}{\text{Min}}\,|y_{c,k}-(2i-\sqrt{M}-1)A| \qquad (7.5.13a)$$

$$l_* = \text{Arg}\,\underset{l=1,\ldots,\sqrt{M}}{\text{Min}}\,\left|y_{s,k}-(2l-\sqrt{M}-1)A\right| \qquad (7.5.13b)$$

to judge the received signal to be the one represented by the (i_*, l_*) th signal point from the left-lower corner in the signal space.

The objective of the following MATLAB program "sim_QAM_passband.m" is to simulate the passband $M=2^b=2^4$-ary QAM signaling depicted in Fig. 7.12 and plot the bit error probability vs. $SNRdB_{r,b}=10\log_{10}(E_b/(N_0/2))$ for checking the validity of theoretical derivation results (7.5.8).



(a) A QAM signal constellation diagram (b) The BER (bit error rate) curve – Bit error probability

Figure 7.13 The signal constellation diagram and BER curve for an $M=2^b=2^{2m}=L^2$-ary QAM signaling

```
%sim_QAM_passband.m
% simulates a digital communication system in Fig.7.13
%  with QAM signal waveforms in Fig.7.11
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
clear, clf
b=4; M=2^b; L=2^(b/2); % # of bits per symbol and the modulation order
SNRbdBt=0:0.1:15;   SNRbt=10.^(SNRbdBt/10);
Pm=2*(1-1/L)*Q(sqrt(3/2*b*SNRbt/(M-1))); % Eq.(7.1.5)
pobet= (1-(1-Pm).^2)/b; % Eq.(7.5.8) with (7.5.6)
Tb=1; Ts=b*Tb;  % Bit/Symbol time
Nb=16; Ns=b*Nb; % # of sample times in Tb and Ts
T=Ts/Ns; LB=4*Ns; LBN1=LB-Ns+1; % Sample time and Buffer size
ssc=[0 0; 0 1; 1 1; 1 0]; sss=ssc;
wc=8*pi/Ts; wcT=wc*T; t=[0:Ns-1]*T;
su=sqrt(2/Ts)*[cos(wc*t); -sin(wc*t)]; suT=su*T; % Basis signals
Esum= 0;
% 16-QAM signal waveforms corresponding to rectangular constellation
for i=1:L
  for l=1:L
    s(i,l,1)=2*i-L-1; s(i,l,2)=2*l-L-1; %In-phase/quadrature amplitude
    Esum= Esum +s(i,l,1)^2 +s(i,l,2)^2;
    ss(L*(l-1)+i,:)=[ssc(i,:) sss(l,:)];
    sw(L*(l-1)+i,:)=s(i,l,1)*su(1,:)+s(i,l,2)*su(2,:);
  end
end
Eav=Esum/M, Es_av=2*(M-1)/3 % Eq.(7.5.4a): Average signal energy (A=1)
Es=2; % Energy of signal waveform
A=sqrt(Es/Eav); sw=A*sw; levels=A*[-(L-1):2:L-1];
SNRdBs=[1:15]; MaxIter=10000; % Range of SNRbdB and # of iterations
for iter=1:length(SNRdBs)
   SNRbdB= SNRdBs(iter);  SNR=10^(SNRbdB/10);
   sigma2=(Es/b)/SNR;  sgmsT=sqrt(sigma2/T);
   yr= zeros(2,LB);  nobe= 0; % Number of bit errors to be accumulated
   for k=1:MaxIter
      im= ceil(rand*L); in= ceil(rand*L);
      imn= (in-1)*L+im; % Index of signal to transmit
      s=ss(imn,:); % Data bits to transmit
      for n=1:Ns   % Operation per symbol time
         wct= wcT*(n-1);  bp_noise= randn*cos(wct)-randn*sin(wct);
         rn= sw(imn,n) + sgmsT*bp_noise;
         yr= [yr(:,2:LB) suT(:,n)*rn]; % Multiplier
      end
      ycsk=sum(yr(:,LBN1:LB)'); % Sampled correlator output - DTR input
      %Detector(DTR)
      [dmin_i,mi]= min(abs(ycsk(1)-levels));
      [dmin_l,ml]= min(abs(ycsk(2)-levels));
      d= ss((ml-1)*L+mi,:); % Detected data bits
      nobe = nobe+sum(s~=d);  if nobe>100; break; end
   end
   pobe(iter)= nobe/(k*b);
end
subplot(222), semilogy(SNRbdBt, pobet, 'k-', SNRdBs, pobe, 'b*')
title('Probability of Bit Error for 16-ary QAM Signaling')
```

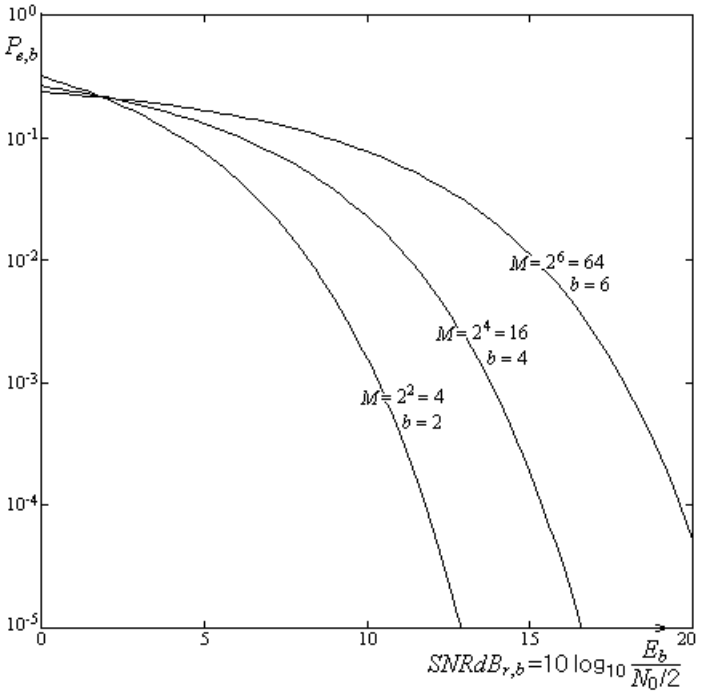


Figure 7.14 The BER curves for $M=2^b$-ary QAM signalings

Fig. 7.14 shows the BER (bit error rate) curves, i.e. the bit error probabilities versus the average SNR per bit for $M = 2^b = 2^2,\ 2^4,\ 2^6$ -ary QAM signalings where the error probability tends to increase as the modulation order M increases. This tendency can be anticipated from the signal constellation diagram where the signal points get denser in the two-dimensional space and consequently, the minimum distance among the signal points gets shorter as M increases.

7.6 COMPARISON OF VARIOUS SIGNALINGS

There are several criteria to consider in deciding the signaling/modulation methods. For example,

- ◊ BER (bit error rate) performance: How low is the bit error probability for the same SNR?
- ◊ Data rate: How high is the data transmission rate[bits/sec]?
- ◊ Power efficiency: How low is the SNR required for keeping the same BER?
- ◊ Bandwidth efficiency: How narrow is the bandwidth required to keep the same data transmission rate?
- ◊ PAR (peak-to-average power ratio), interference, and out-of-band radiation
- ◊ Structural simplicity and cost: How simple is the structure and how cheap is the cost for construction and maintenance?

Table 7.1 shows the BERs for various signalings that have been discussed so far. The following MATLAB routine "prob_error(SNRbdB,signaling,b)" computes the error probability for SNRbdB value(s), signaling method, and number of bits per symbol. The MATLAB built-in function 'berawgn(EbN0dB,signaling,M)' is more powerful and convenient to use. The BERTool GUI (graphic user interface) can be invoked by typing 'bertool' into the MATLAB Command Window.

Table 7.1 Bit error probabilities for various signalings ($SNR_{r,b} = E_b/\sigma^2 = (E_s/b)/(N_0/2)$)

$M=2^b$	Signaling	Coherent (synchronous) detection	Noncoherent detection
Binary case $M=2^b$ ($b=1$)	OOK	$Q\left(\sqrt{\frac{E_s/4}{N_0/2}}\right) = Q\left(\sqrt{\frac{SNR_r}{4}}\right)$ (7.1.24)	$\frac{1}{2}e^{-SNR_r/4}$ (7.1.22)
	FSK	$Q\left(\sqrt{\frac{E_s/2}{N_0/2}}\right) = Q\left(\sqrt{\frac{SNR_r}{2}}\right)$ (7.2.9)	$\frac{1}{2}e^{-SNR_r/4}$ (7.2.20)
	PSK	$Q\left(\sqrt{\frac{E_s}{N_0/2}}\right) = Q\left(\sqrt{SNR_r}\right)$ (7.3.5)	DPSK: $\frac{1}{2}e^{-SNR_r/2}$ (7.4.9)
$M=2^b$ -ary ($b>1$)	ASK	$\frac{2(M-1)}{bM}Q\left(\sqrt{\frac{3b\,SNR_{r,b}}{M^2-1}}\right)$ (7.1.5/6)	-
	FSK	$\frac{M/2}{M-1}\left\{1-\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}q(y)e^{-y^2}dy\right\}$ (7.2.8/9) with $q(y)=Q^{M-1}\left(-\sqrt{2}y-\sqrt{b\,SNR_{r,b}}\right)$	$\frac{M/2}{M-1}\sum_{m=0}^{M-1}(-1)^{m+1}\binom{M-1}{m}\frac{1}{m+1}e^{-\frac{mb\,SNR_r}{2(m+1)}}$ (7.2.19)
	PSK	$\frac{2}{b}Q\left(\sqrt{b\,SNR_{r,b}}\sin\frac{\pi}{M}\right)$ (7.3.8)	$\frac{2}{b}Q\left(\sqrt{b\,SNR_{r,b}/2}\sin\frac{\pi}{M}\right)$ (7.4.8)
	QAM	$\le\frac{4(L-1)}{bL}Q\left(\sqrt{\frac{3b/2}{M-1}SNR_{r,b}}\right)$ (7.5.8) with $M=LN(L>N)$	-

Now, let us look over the *bandwidth efficiency*, which is defined to be the ratio of the data bit (transmission) rate R_b [bits/s] over the required system bandwidth B [Hz], for $M=2^b$ -ary ASK/PSK/QAM/FSK signalings. Note the following:

- The rectangular pulse of duration D [s] in Fig. 1.1(a1) can be viewed as a signal carrying the data with symbol rate $1/D$ [symbol/s] and the null-to-null bandwidth of the rectangular pulse is $4\pi/D$ [rad/s] $= 2/D$ [Hz] as can be seen from the spectrum in Fig. 1.1(a1). Thus it may be conjectured that, if a series of data with symbol rate R_s [symbol/s] is modulated with a carrier frequency ω_c and transmitted, the bandwidth is $2R_s$ [Hz].
- The interval between adjacent discrete spectra in Fig. 1.1(a1) is π/D [rad/s] $=1/2D$ [Hz], which implies that the gap between the carrier frequencies in FSK signaling is $R_s/2$ [Hz].

Therefore we can write the bandwidths and bandwidth efficiencies of ASK/PSK/QAM and (coherent) FSK signalings as

$$B_{ASK/PSK/QAM} = 2R_s = \frac{2}{b}R_b = \frac{2R_b}{b=\log_2 M}; \quad \frac{R_b}{B_{ASK/PSK/QAM}} = \frac{b=\log_2 M}{2} \tag{7.6.1}$$

$$B_{coh,FSK} = 2R_s + (M-1)\frac{R_s}{2} = \frac{(M+3)R_b/\log_2 M}{2}; \quad \frac{R_b}{B_{coh,FSK}} = \frac{2\log_2 M}{M+3} \tag{7.6.2}$$

```
function p=prob_error(SNRbdB,signaling,b,opt1,opt2)
% Finds the symbol/bit error probability for given SNRbdB (Table 7.1)
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
if nargin<5, opt2='coherent'; end  % opt2='coherent' or 'noncoherent'
if nargin<4, opt1='SER'; end % opt1='SER' or 'BER'
M=2^b; SNRb=10.^(SNRbdB/10);  NSNR=length(SNRb);
if signaling(1:3)=='ASK' % ASK (PAM)
  if lower(opt2(1))=='c' % ASK coherent --> Eq.(7.1.5)
    for i=1:NSNR, p(i)=2*(M-1)/M*Q(sqrt(3*b*SNRb(i)/(M^2-1))); end
    if lower(opt1(1))=='b', p = p/b; end
   else  % ASK noncoherent -->  Eq.(7.1.22)
    if b==1,  for i=1:NSNR, p(i)=exp(-SNRb(i)/4)/2; end;  end
  end
elseif signaling(1:3)=='FSK'
  tmp=M/2/(M-1);
  f5251_=inline('Q(-sqrt(2)*x-sqrt(b*SNRb)).^(2^b-1)','x','SNRb','b');
  if lower(opt2(1))=='c' % FSK coherent
    if b==1
      for i=1:NSNR, p(i)=Q(sqrt(SNRb(i)/2)); end   %Eq.(7.2.9)
     else
      for i=1:NSNR
         p(i) = 1-Gauss_Hermite(f5251_,10,SNRb(i),b)/sqrt(pi);
      end
    end
  else % FSK noncoherent
    for i=1:NSNR
       p(i)=(M-1)/2*exp(-b*SNRb(i)/4);  tmp1=M-1;
       for m=2:M-1
          tmp1=-tmp1*(M-m)/m;
          p(i)=p(i)+tmp1/(m+1)*exp(-m*b*SNRb(i)/2/(m+1)); % Eq.(7.2.19)
       end
    end
  end
  if lower(opt1(1))=='b'&b>1, p = p*tmp; end
elseif signaling(1:3)=='PSK'  % Eq.(7.3.7)
  for i=1:NSNR, p(i)=(1+(b>1))*Q(sqrt(b*SNRb(i))*sin(pi/M));  end
  if lower(opt1(1))=='b'&b>1, p = p/b; end
elseif signaling(1:3)=='DPS' % DPSK
  if b==1  % Eq.(7.4.8)
    for i=1:NSNR, p(i)=2*Q(sqrt(b*SNRb(i)/2)*sin(pi/M)); end
   else
    for i=1:NSNR, p(i)=exp(-SNRb(i)/2)/2; end %Eq.(7.4.9)
    if lower(opt1(1))=='b', p = p/b; end
  end
 elseif signaling(1:3)=='QAM'
  L=2^(ceil(b/2)); N = M/L;
  for i=1:NSNR
     tmpL = 1-2*(L-1)/L*Q(sqrt(3*b/2/(L^2-1)*SNRb(i)));
     tmpN = 1-2*(N-1)/N*Q(sqrt(3*b/2/(N^2-1)*SNRb(i)));
     p(i) = 1-tmpL*tmpN; % Eq.(7.5.6)
  end
  if lower(opt1(1))=='b'&b>1, p = p/b; end
end
```

Table 7.2 Power efficiency and bandwidth efficiency with signaling

The number of bits per symbol	<Power efficiency> $SNRdB_{r,b} = 10\log_{10}(E_b/(N_0/2))$ [dB] for $P_{e,b} = 10^{-5}$				<Bandwidth efficiency> R_b / B [bits/s/Hz]		
	ASK	PSK	FSK	QAM	ASK/PSK	FSK	QAM
$b = 1$	12.6	12.6	15.6	-	0.5000	0.4000	-
$b = 2$	16.8	12.9	13.1	12.9	1.0000	0.5714	1.0000
$b = 3$	21.3	16.5	11.6	-	1.5000	0.5455	-
$b = 4$	26.1	21.1	10.7	17.1	2.0000	0.4211	2.0000
$b = 5$	31.2	26.1	9.9	-	2.5000	0.2857	-
$b = 6$	36.5	31.3	9.4	21.6	3.0000	0.1791	3.0000
$b = 7$	41.8	36.7	8.9	-	3.5000	0.1069	-
$b = 8$	47.3	42.1	8.5	26.4	4.0000	0.0618	4.0000

```
%dc07t02.m
% fills in Table 7.2 with SNRdB for BER=1e-5 and bandwidth efficiency
clear, clf
tol=1e-14; bs=[1:8];
% Define a nonlinear equation to be solved for SNRdB using fsolve()
nonlinear_eq=inline('prob_error(x,signaling,b)-1e-5','x','signaling','b');
% For PSK signaling
for i=1:length(bs)
   b=bs(i); RB_PSK(i)=b/2; % Bandwidth efficiency
   if i==1, x0=10;  else  x0=SNRbdBs_PSK(i-1);  end
   SNRbdBs_PSK(i)=fsolve(nonlinear_eq,x0,optimset('TolFun',tol),'PSK',b);
end
disp('PSK'), [bs; SNRbdBs_PSK; RB_PSK]
% For FSK signaling
for i=1:length(bs)
   b=bs(i); RB_FSK(i)=2*b/(2^b+3);
   if i==1, x0=10;  else  x0=SNRbdBs_FSK(i-1);  end
   SNRbdBs_FSK(i)=fsolve(nonlinear_eq,x0,optimset('TolFun',tol),'FSK',b);
end
disp('FSK'), [bs; SNRbdBs_FSK; RB_FSK]
% For QAM signaling
bs_QAM=[2:2:8];
for i=1:length(bs_QAM)
   b=bs_QAM(i); RB_QAM(i)=b/2;
   if i==1, x0=10;  else  x0=SNRbdBs_QAM(i-1);  end
   SNRbdBs_QAM(i)=fsolve(nonlinear_eq,x0,optimset('TolFun',tol),'QAM',b);
end
disp('QAM'), [bs_QAM; SNRbdBs_QAM; RB_QAM]
% For ASK signaling
for i=1:length(bs)
   b=bs(i); RB_ASK(i)=b/2;
   if i==1, x0=10;  else  x0=SNRbdBs_ASK(i-1);  end
   SNRbdBs_ASK(i)=fsolve(nonlinear_eq,x0,optimset('TolFun',tol),'ASK',b);
end
disp('ASK'), [bs; SNRbdBs_ASK; RB_ASK]
```

Table 7.2 shows the *power efficiency*, i.e. the SNR[dB] required to achieve the BER of 10^{-5} and the *bandwidth efficiency*, i.e. the ratio of data transmission rate[bits/s] over bandwidth[Hz], that are also depicted in Fig. 9.7. As the modulation order M or the number b of bits per symbol increases, FSK tends to have higher power efficiency and lower bandwidth efficiency while PSK tends to have lower power efficiency and higher bandwidth efficiency, as can also be seen from Figs. 7.5 and 7.10. The above MATLAB program "dc07t02.m" can be used to compute the values of $SNRdB_{r,b}$ at which PSK/FSK/QAM/ASK signalings achieve the BER of 10^{-5}.

Before ending this section, it would be well worth to see the usage of the BER computing function 'berawgn()' in the MATLAB Help manual:

```
ber = berawgn(EbNo,'PAM',M)
Ber = berawgn(EbNo,'QAM',M)
Ber = berawgn(EbNo,'PSK',M,dataenc)  with dataenc='diff'/'nondiff' for differential/non-differential
ber = berawgn(EbNo,'OQPSK',dataenc)  for OQPSK (offset QPSK)  (see Problem 7.4)
ber = berawgn(EbNo,'DPSK',M)
ber = berawgn(EbNo,'FSK',M,coh)  with coh='coherent'/'noncoherent' for coherent/noncoherent
ber = berawgn(EbNo,'FSK',2,coh,rho)  with rho=the complex correlation coefficient
ber = berawgn(EbNo,'MSK',dataenc)  with dataenc='diff'/'nondiff' for differential/non-differential
ber = berawgn(EbNo,'MSK',dataenc,coherence)
berlb = berawgn(EbNo,'CPFSK',M,modindex,kmin) % gives the BER lowerbound
```

Problems

7.1 Linear Combination of Gaussian Noises

Suppose we have a zero-mean white Gaussian noise $n(t)$ of mean and variance as

$$E\{n(t)\} = 0 \tag{P7.1.1}$$

$$\text{cov}\{n(\tau), n(t)\} = E\{n(\tau)n(t)\} = \sigma^2\delta(\tau - t) \tag{P7.1.2}$$

Also, consider another noise

$$n_1(t) = \int_0^{T_s} n(\tau + t)s_u(\tau)\, d\tau \tag{P7.1.3}$$

which is obtained from taking the crosscorrelation of a zero-mean white noise $n(t)$ with a unit energy signal $s_u(t)$ such that

$$\int_0^{T_s} s_u^2(\tau)\, d\tau = 1 \tag{P7.1.4}$$

It is claimed that this is also a zero-mean Gaussian noise with variance σ^2.

(a) To verify this, check the following derivation of the mean and covariance of $n_1(t)$. Note that a linear combination of Gaussian processes is also Gaussian.

```
%dc07p01.m
% See if a linear convolution of Gaussian noises
% with a unit signal waveform is another Gaussian noise
clear, clf
K=10000; % # of iterations for getting the error probability
Ts=1; N=40; T=Ts/N; % Symbol time and Sample time
N4=N*4; % Buffer size of correlator
wc=10*pi/Ts;  t=[0:N-1]*T;  wct=wc*t;
su=sqrt(2/Ts)*cos(wct); % Unit energy signal
signal_power=su*su'*T % Signal energy
sigma2=2; sigma=sqrt(sigma2); sqT=sqrt(T);
noise= zeros(1,N4); % Noise buffer
for k=1:K
   for n=1:N % Operation per symbol time
      noise0= sigma*randn;
      noise=[noise(2:N4) noise0/sqT]; % Bandpass noise
      noise1= su*noise(3*N+1:N4)'*T;
   end
   noise0s(k)=noise0; noise1s(k)=noise1;
end
phi0=xcorr1(noise0s);  phi1=xcorr1(noise1s);
plot(phi0), hold on, pause, plot(phi1,'r')
```

(proof)

$$E\{n_1(t)\} = E\left\{\int_0^{T_s} n(\tau+t)s_u(\tau)\,d\tau\right\} = \int_0^{T_s} E\{n(\tau+t)\}s_u(\tau)\,d\tau \overset{(P7.1.1)}{=} 0 \qquad \text{(P7.1.5)}$$

$$E\{n_1^2(t)\} = E\left\{\int_0^{T_s} n(\tau+t)s_u(\tau)d\tau \int_0^{T_s} n(v+t)s_u(v)dv\right\}$$

$$= \int_0^{T_s}\int_0^{T_s} s_u(\tau)s_u(v)\,E\{n(\tau+t)n(v+t)\}\,d\tau\,dv$$

$$\overset{(P7.1.2)}{=} \sigma^2\int_0^{T_s}\int_0^{T_s} s_u(\tau)s_u(v)\,\delta(\tau-v)\,d\tau\,dv \overset{(E1.6.1)}{=} \sigma^2\int_0^{T_s} s_u^2(\tau)\,d\tau \overset{(P7.1.4)}{=} \sigma^2 \qquad \text{(P7.1.6)}$$

(b) As an alternative for verification, the above program "dc07p01.m" is composed to generate a noise $n(t)$ with Gaussian distribution $N(m=0, \sigma^2=2)$ and take the sampled crosscorrelation between $n(t)$ and a unit energy signal $s_u(t)$ to make another noise $n_1(t)$. Plot the autocorrelation of these two noises (`noise0` and `noise1`) and make a comment on their closeness or similarity of their statistical properties such as the mean and variance.

7.2 Coherent/Noncoherent Detection with Time Difference between XMTR and RCVR

(a) In order to feel how seriously the time difference between XMTR and RCVR affects the BER performance of a communication system, use the following statements to modify the MATLAB program "sim_FSK_passband_coherent.m" so that it can accommodate some delay time or time difference (`td`) between XMTR and RCVR clocks. Then run the modified program to see the changed BER curve. Does it stay away from the theoretical BER curve?

```
nd=2; % Number of delay samples
sws=zeros(1,LB);
sws=[sws(2:LB) sw(i,n)];
r=[r(2:LB) sws(end-nd)+sgmsT*bp_noise]; % Received signal
```

(b) In order to see that noncoherent detection is of relative advantage over coherent detection in facing the time diffference between XMTR and RCVR, replace `nd=0` by `nd=2` in the 14th line of the program "sim_FSK_passband_noncoherent.m" (Sec. 7.2) and run the modified program to see the changed BER curve. Does it still touch the theoretical BER curve?

(c) Modify the 19th line of the QPSK simulation program "sim_PSK_passband.m" (Sec. 7.3) into nd=1 and run the program to see the changed constellation diagram and BER curve. Modify the 16th line of the QDPSK simulation program "sim_DPSK_passband.m" (Sec. 7.4) into nd=1 and run the program to see the BER curve. What is implied by the simulation results?

7.3 $M=2^2$-ary QAM (Quadrature AM) and $\pi/4$-QPSK(Quadrature PSK)

From the QAM signal constellation diagrams in Fig. 7.11, it can be seen that $M=2^2$-ary QAM is the same as $\pi/4$-QPSK, whose signal constellation diagram is a rotation of the QPSK signal constellation diagram (Fig. 7.1(b3)) by $\pi/4=45^o$ as depicted in Fig. P7.5(a2). Is it also supported by the conformity of the error probabilities (between the standard QPSK and the $\pi/4$-QPSK) that turned out to be Eq. (7.3.7) and Eq. (7.5.7), respectively? You can substitute $b=2$, $L=2^{b/2}=2$, and $M=2^2$ into both of the equations and check if they conform with each other.

7.4 OQPSK (Offset Quadrature PSK) or SQPSK (Staggered QPSK)

As depicted in Fig. P7.4, OQPSK is a slight modification of QPSK (Fig. 7.12) in such a way that the Q(uadrature)-channel bit stream is offset w.r.t. the I(n-phase)-channel by a bit duration (Fig. P7.4(b2)). Compared with QPSK, it allows no simultaneous change of two bits to prevent any state transition accompanying the phase change of 180^{o} (Fig. P7.4(c2)) (in the teeth of making more frequent state transitions possibly every bit time) and consequently, the envelope variation becomes less severe so that the transmitted signal bandwidth can be limited more strictly.

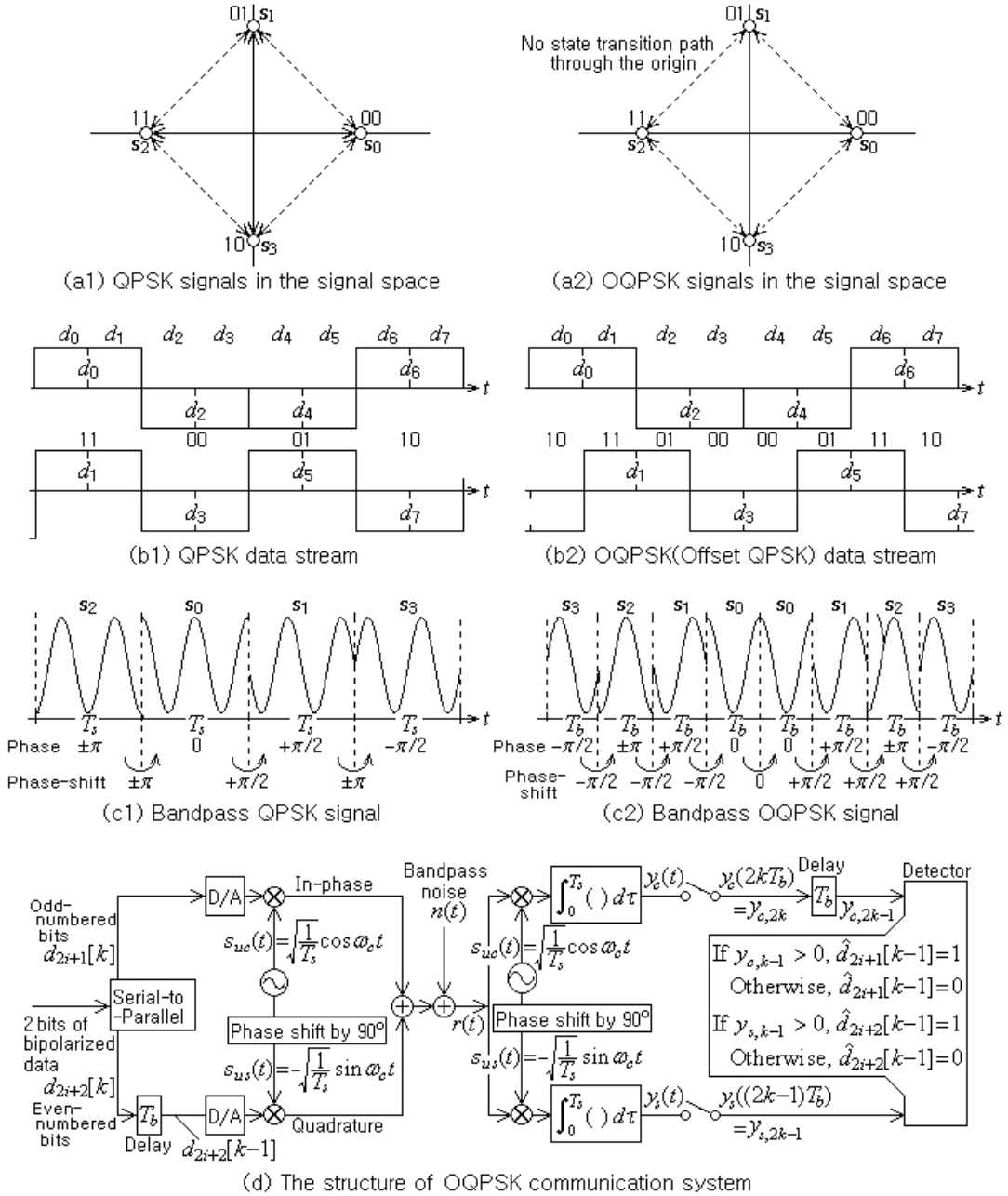
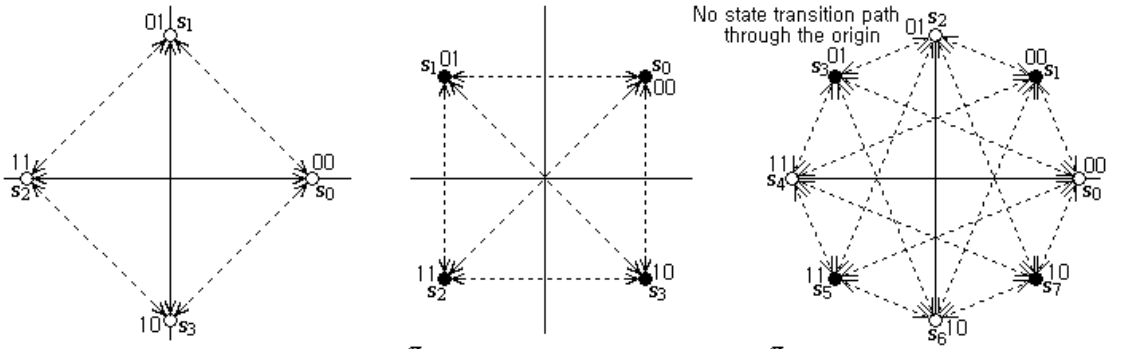


Figure P7.4 QPSK and OQPSK communication systems

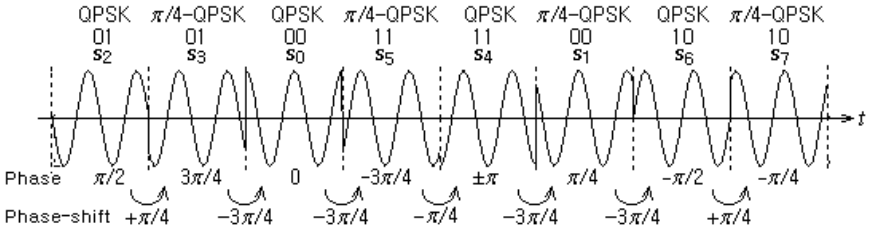
```
%sim_OQPSK.m
% simulates a digital communication system
%  with O(ffset)QPSK signal waveforms in Fig.P7.4
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
clear, clf
b=2; M=2^b;
SNRbdBt=0:0.1:10;  SNRbt=10.^(SNRbdBt/10);
pobet=prob_error(SNRbdBt,'PSK',b,'bit');
Tb=1; Ts=b*Tb; % Bit/symbol time
Nb=16; Ns=Nb*b; % # of sample times in Tb and Ts
T=Ts/Ns; LB=4*Ns; LBN1=LB-Ns+1; % Sample time and Buffer size
Es=2; sqEb=sqrt(Es/b); % Energy of signal waveform
% QPSK signal waveforms
ss=[0 0; 0 1; 1 1; 1 0];
wc=2*pi/Ts; wcT=wc*T; t=[0:Ns-1]*T;
su=sqrt(2/Ts)*[cos(wc*t); ??????????];  suT=su*T;
sw=sqEb*su;
SNRdBs=[1:3:10]; MaxIter=10000; % Range of SNRdB, # of iterations
for iter=1:length(SNRdBs)
   SNRbdB=SNRdBs(iter);  SNRb=10^(SNRbdB/10);
   N0=2*(Es/b)/SNRb; sigma2=N0/2; sgmsT=sqrt(sigma2/T);
   yr= zeros(2,LB); yc=zeros(1,2); ys=zeros(1,2);
   iq=0; % Initialize the quadrature bit arbitrarily
   nobe=0; % Number of bit errors to be accumulated
   for k=1:MaxIter
      i=ceil(rand*M); s=ss(i,:);  wct=-wcT;
      for n=1:b % Operation per symbol time
         if n==1, ii=2*ss(i,1)-1; % In-phase bit
          else iq=2*ss(i,2)-1; % Quadrature bit
         end
         mn=0;
         for m=1:Nb % Operation per bit time
            wct= wct+wcT; mn=mn+1;
            bp_noise= randn*cos(wct)-randn*sin(wct);
            rn=ii*sw(1,mn)+iq*sw(2,mn)+sgmsT*bp_noise;
            yr=[yr(:,2:LB) suT(:,mn)*rn]; % Multiplier
         end
         if n==2  % sampled at t=2k*Tb
           yc=[yc(2) sum(yr(?,LBN1:LB))]; % Correlator output
          else  % sampled at t=(2k-1)*Tb
           ys=[ys(2) sum(yr(?,LBN1:LB))]; % Correlator output
         end
      end
      d=([yc(?) ys(?)]>0); % Detector(DTR)
      if k>1, nobe=nobe+sum(s0~=d); end
      if nobe>100, break; end
      s0= s;
   end
   pobe(iter)= nobe/(k*b);
end
pobe
semilogy(SNRbdBt,pobet,'k-', SNRdBs,pobe,'b*')
title('Probability of Bit Error for (4-ary) QPSK Signaling')
```

The above MATLAB program "sim_OQPSK.m" is a modification of the program "sim_PSK_passband.m" (QPSK) (in Sec. 7.3) to simulate the OQPSK communication system, whose block diagram is depicted in Fig. P7.4, but it is unfinished. Finish it up by replacing the three parts of ?'s with appropriate statements or just indices and run it to see the BER curve.

7.5 $\pi/4$-Shifted QPSK (Quadrature PSK)



(a1) QPSK signal constellation (a2) $\frac{\pi}{4}$-QPSK signal constellation (a3) $\frac{\pi}{4}$-shifted QPSK signal constellation



(b) A data stream and the corresponding bandpass $\frac{\pi}{4}$-shifted QPSK signal

Figure P7.5 $\frac{\pi}{4}$-shifted QPSK (Quadrature Phase Shift Keying) signaling

$\pi/4$-shifted QPSK is another way of lowering envelope fluctuations than OQPSK discussed in Problem 7.4. It assigns one of the signal points in the (non-offset) QPSK (Fig. P7.5(a1)) and $\pi/4$-QPSK (Fig. P7.5(a2)) signal constellations to even and odd-number indexed data symbols as depicted in Fig. P7.5(b) and therefore, it virtually uses the dual signal constellation diagram shown in Fig. P7.5(a3) where the modulation causes no state transition path through the origin as in OQPSK. Compared with OQPSK, it has the maximum phase change of $\pm 3\pi/4$, which is larger than that ($\pm\pi/2$) of OQPSK; On the other hand, differential encoding/decoding with noncoherent detection can be implemented in $\pi/4$-shifted QPSK signaling, since it has four possible phase-shifts of $\Delta\theta = \pm\pi/4$ and $\pm 3\pi/4$ (as can be seen from Fig. P7.5(b)) while OQPSK has only two possible phase-shifts of $\Delta\theta = \pm\pi/2$.

Table P7.5 Message data dibits and the corresponding phase-shifts in $\pi/4$-shifted QDPSK

(Gray-coded) message dibits	Phase-shifts	$s_k = s_0\ s_1\ s_2\ s_3\ s_4\ s_5\ s_6\ s_7$
$d_{2k-1}d_{2k} = 00$	$\Delta\theta = +\pi/4$	$s_{(k+1) \bmod 8} = s_1\ s_2\ s_3\ s_4\ s_5\ s_6\ s_7\ s_0$
$d_{2k-1}d_{2k} = 01$	$\Delta\theta = +3\pi/4$	$s_{(k+3) \bmod 8} = s_3\ s_4\ s_5\ s_6\ s_7\ s_0\ s_1\ s_2$
$d_{2k-1}d_{2k} = 11$	$\Delta\theta = -3\pi/4$	$s_{(k+5) \bmod 8} = s_5\ s_6\ s_7\ s_0\ s_1\ s_2\ s_3\ s_4$
$d_{2k-1}d_{2k} = 10$	$\Delta\theta = -\pi/4$	$s_{(k+7) \bmod 8} = s_7\ s_0\ s_1\ s_2\ s_3\ s_4\ s_5\ s_6$

(a) Modify the (standard) QPSK simulation program "sim_PSK_passband.m" into a $\pi/4$-shifted QPSK simulation program named, say, "sim_S_QPSK.m" and run it to see the BER curve. Does it conform with the theoretical BER curve?

(b) Referring to Table P7.5, modify the (standard) QDPSK (quadrature differential phase shift keying) simulation program "sim_DPSK_passband.m" into a $\pi/4$-shifted QDPSK simulation program named, say, "sim_S_QDPSK.m" and run it to see the BER curve. Does it conform with the theoretical one? If you have no idea, start with the following unfinished program:

```
%sim_S_QDPSK.m
% simulates the pi/4-shifted QDPSK signaling (Table P7.5)
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
b=2; M=2^b; M2=M*2;
SNRbdBt=0:0.1:10;   SNRbt=10.^(SNRbdBt/10);
pobet=prob_error(SNRbdBt,'DPSK',b,'bit');
Tb=1; Ts=b*Tb; % Bit/symbol time
Nb=16; Ns=b*Nb; % Numbers of sample times in Tb and Ts
T=Ts/Ns; LB=4*Ns; LBN1=LB-Ns+1; % Sample time and Buffer size
Es=2; % Energy of signal waveform
% QDPSK signal waveforms
ss=[0 0; 0 1; 1 1; 1 0];
wc=8*pi/Ts; wcT=wc*T; t=[0:Ns-1]*T;  nd=1;
for m=1:M2,  sw(m,:)=sqrt(2*Es/Ts)*cos(wc*t+(m-1)*pi/M);  end
su= sqrt(2/Ts)*[cos(wc*t); -sin(wc*t)]; suT=su*T;
SNRdBs =[1:3:10]; MaxIter=10000; %Range of SNRbdB, # of iterations
for iter=1:length(SNRdBs)
   SNRbdB=SNRdBs(iter);  SNRb=10^(SNRbdB/10);
   N0=2*(Es/b)/SNRb; sigma2=N0/2; sgmsT=sqrt(sigma2/T);
   sws=zeros(1,LB); yr=zeros(2,LB);
   nobe=0; % Number of bit errors to be accumulated
   is0=1; % Initial signal index
   th0=1; % Initial guess (possibly wrong)
   for k=1:MaxIter
      i= ceil(rand*M); s=ss(i,:); % Data bits to transmit
      is= mod(is0+?????,M2)+1; % Signal to transmit (Table P7.5)
      for n=1:Ns % Operation per symbol time
         sws=[sws(2:LB) sw(is,n)];
         wct=wcT*(n-1); bp_noise= randn*cos(wct)-randn*sin(wct);
         rn= sws(end-nd) + sgmsT*bp_noise;
         yr=[yr(:,2:LB) suT(:,n)*rn]; % Multiplier
      end
      ycsk=sum(yr(:,LBN1:LB)')'; % Sampled correlator output
      %Detector(DTR)
      th=atan2(ycsk(2),ycsk(1)); dth=th-th0;
      if dth<0, dth=dth+2*pi; end
      [themin,lmin]=min(abs(dth-[?????]*2*pi/M));
      d= ss(lmin,:); % Detected data bits
      nobe= nobe+sum(s~=d);  if nobe>100, break; end
      is0=is; th0=th; % update the previous signal and theta
   end
   pobe(iter)= nobe/(k*b);
end
semilogy(SNRbdBt,pobet,'k', SNRdBs,pobe,'b*')
```

7.6 Minimum-Shift Keying (MSK)

Minimum-shift keying is a type of continuous phase frequency shift keying (CP-FSK). It encodes each data bit into a half sinusoid

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(\omega_c t + \theta(t)) \tag{P7.6.1}$$

with its phase changed continuously as

$$\theta(t) = \theta(kT_b) \pm \frac{\pi}{2T_b}(t - kT_b) \quad \text{with } \theta(0) = 0 \tag{P7.6.2}$$

where the second term has a plus or minus sign for each bit interval depending on whether the value of message data bit is 1 or 0 (see Fig. P7.6). As a consequence, the phase of the signal waveform at the boundaries between two consecutive bit intervals becomes

$$\theta(2kT_b) = \pm m\pi \text{ or } \theta((2k+1)T_b) = \pm\frac{\pi}{2} \pm m\pi \tag{P7.6.3}$$

and the instantaneous frequency, which is obtained by differentiating the angular argument of $s(t)$ w.r.t. t, becomes

$$\omega_1 = \omega_c + \frac{\pi}{2T_b} \quad \text{or} \quad \omega_0 = \omega_c - \frac{\pi}{2T_b} \tag{P7.6.4}$$

Note that the difference of these two frequencies is π / T_b and it is the same as the minimum frequency gap required to keep the orthogonality between two frequency components for a bit duration T_b (see the discussion just below Eq. (7.2.4)).

To find two (orthogonal) basis signal waveforms that can be used to detect the phase of the received signal, we rewrite Eq. (P7.6.1) as

$$s(t) \overset{\text{(P7.6.1)}}{\underset{\text{(D.20)}}{=}} \sqrt{\frac{2E_b}{T_b}} \left(\cos\theta(t)\cos\omega_c t - \sin\theta(t)\sin\omega_c t\right) = s_I(t)\cos\omega_c t - s_Q(t)\sin\omega_c t \tag{P7.6.5}$$

where

$$\begin{aligned}
s_I(t) &= \sqrt{\frac{2E_b}{T_b}}\cos\theta(t) \quad \text{with } \theta(t) \overset{\text{(P7.6.2)}}{=} \theta(2kT_b) \pm \frac{\pi}{2T_b}(t - 2kT_b) \\
&\overset{\text{(D.20)}}{=} \sqrt{\frac{2E_b}{T_b}}\left(\cos\theta(2kT_b)\cos\left(\pm\frac{\pi}{2T_b}(t-2kT_b)\right) - \sin\theta(2kT_b)\sin\left(\pm\frac{\pi}{2T_b}(t-2kT_b)\right)\right) \\
&= \sqrt{\frac{2E_b}{T_b}}\cos\theta(2kT_b)\cos\left(\pm\frac{\pi}{2T_b}t\right) \quad \text{for } (2k-1)T_b < t \le (2k+1)T_b \\
&= \begin{cases} +\sqrt{\dfrac{2E_b}{T_b}}\cos\left(\dfrac{\pi}{2T_b}t\right) & \text{if } \theta(2kT_b) = 0 \\ -\sqrt{\dfrac{2E_b}{T_b}}\cos\left(\dfrac{\pi}{2T_b}t\right) & \text{if } \theta(2kT_b) = \pm\pi \end{cases}
\end{aligned} \tag{P7.6.6a}$$

$$s_Q(t) = \sqrt{\frac{2E_b}{T_b}} \sin\theta(t) \quad \text{with } \theta(t) \overset{\text{(P7.6.2)}}{=} \theta((2k+1)T_b) \pm \frac{\pi}{2T_b}(t-(2k+1)T_b)$$

$$\overset{\text{(D.19)}}{=} \sqrt{\frac{2E_b}{T_b}} \begin{pmatrix} \sin\theta((2k+1)T_b)\cos\left(\pm\frac{\pi}{2T_b}(t-(2k+1)T_b)\right) \\ \pm\cos\theta((2k+1)T_b)\sin\left(\pm\frac{\pi}{2T_b}(t-(2k+1)T_b)\right) \end{pmatrix}$$

$$= \sqrt{\frac{2E_b}{T_b}} \sin\theta((2k+1)T_b)\sin\left(\pm\frac{\pi}{2T_b}t\right) \quad \text{for } 2kT_b < t \le 2(k+1)T_b$$

$$= \begin{cases} +\sqrt{\frac{2E_b}{T_b}}\sin\left(\frac{\pi}{2T_b}t\right) & \text{if } \theta((2k+1)T_b) = +\frac{\pi}{2} \\ -\sqrt{\frac{2E_b}{T_b}}\sin\left(\frac{\pi}{2T_b}t\right) & \text{if } \theta((2k+1)T_b) = -\frac{\pi}{2} \end{cases} \tag{P7.6.6b}$$

Substituting these two equations (P7.6.6a) and (P7.6.6b) into Eq. (P7.6.5) yields

$$s(t) = \pm\sqrt{\frac{2E_b}{T_b}}\cos\left(\frac{\pi}{2T_b}t\right)\cos\omega_c t \mp \sqrt{\frac{2E_b}{T_b}}\sin\left(\frac{\pi}{2T_b}t\right)\sin\omega_c t \tag{P7.6.7}$$

This implies that the basis signal waveforms to be used for detection at RCVR are

$$s_{uc}(t) = \sqrt{\frac{2}{T_b}}\cos\left(\frac{\pi}{2T_b}t\right)\cos\omega_c t \tag{P7.6.8a}$$

$$s_{us}(t) = \sqrt{\frac{2}{T_b}}\sin\left(\frac{\pi}{2T_b}t\right)\sin\omega_c t \tag{P7.6.8b}$$

Therefore, RCVR computes the correlation between the received signal $r(t)$ and these two basis signal waveforms and samples it at $t = 2kT_b$ and $(2k+1)T_b$ to get

$$y_c((2k+1)T_b) \overset{\text{(5.1.27)}}{=} \int_{-T_b}^{T_b} s_{uc}(t)r(t+2kT_b)\,dt = \int_{-T_b}^{T_b} s_{uc}(t)s(t+2kT_b)\,dt + n_c$$

$$\overset{\text{(P7.6.1)}}{\underset{\text{(P7.6.8a)}}{=}} \sqrt{\frac{2}{T_b}}\sqrt{\frac{2E_b}{T_b}}\int_{-T_b}^{T_b}\cos\left(\frac{\pi}{2T_b}t\right)\cos\omega_c t\cos(\omega_c t+\theta(2kT_b))\,dt + n_c$$

$$\overset{\text{(D.25)}}{=} \frac{\sqrt{E_b}}{T_b}\int_{-T_b}^{T_b}\cos\left(\frac{\pi}{2T_b}t\right)\left(\cos(2\omega_c t+\theta(2kT_b))+\cos\theta(2kT_b)\right)\,dt + n_c \tag{P7.6.9a}$$

$$\overset{\text{(D.25)}}{=} \frac{\sqrt{E_b}}{2T_b}\int_{-T_b}^{T_b}\begin{pmatrix} \cos\left(2\omega_c t+\theta(2kT_b)+\frac{\pi}{2T_b}t\right)+\cos\left(2\omega_c t+\theta(2kT_b)-\frac{\pi}{2T_b}t\right) \\ +\cos\left(\theta(2kT_b)+\frac{\pi}{2T_b}t\right)+\cos\left(\theta(2kT_b)-\frac{\pi}{2T_b}t\right) \end{pmatrix} dt + n_c$$

$$= \sqrt{E_b}\cos(\theta(2kT_b)) + n_c$$

$$y_s(2kT_b) \overset{(5.1.27)}{=} \int_0^{2T_b} s_{us}(t)\, r(t+(2k-1)T_b)\, dt = \int_0^{2T_b} s_{us}(t)\, s(t+(2k-1)T_b)\, dt + n_s$$

$$\overset{\substack{(P7.6.1)\\(P7.6.8b)}}{=} \sqrt{\frac{2E_b}{T_b}}\sqrt{\frac{2}{T_b}}\int_0^{2T_b} \sin\left(\frac{\pi}{2T_b}t\right)\sin\omega_c t\cos(\omega_c t+\theta((2k-1)T_b))\, dt + n_s$$

$$\overset{(D.24)}{=} \frac{\sqrt{E_b}}{T_b}\int_0^{2T_b} \sin\left(\frac{\pi}{2T_b}t\right)\left(\sin(2\omega_c t+\theta((2k-1)T_b))-\sin\theta((2k-1)T_b)\right)\, dt + n_s \qquad \text{(P7.6.9b)}$$

$$\overset{(D.22)}{=} \frac{\sqrt{E_b}}{2T_b}\int_0^{2T_b} \left(\begin{array}{l}\cos\left(2\omega_c t+\theta((2k-1)T_b)-\frac{\pi}{2T_b}t\right)-\cos\left(2\omega_c t+\theta((2k-1)T_b)+\frac{\pi}{2T_b}t\right)\\ +\cos\left(\theta((2k-1)T_b)+\frac{\pi}{2T_b}t\right)-\cos\left(\theta((2k-1)T_b)-\frac{\pi}{2T_b}t\right)\end{array}\right) dt + n_s$$

$$= -\sqrt{E_b}\sin(\theta((2k-1)T_b)) + n_s$$

Depending on the sign of these sampled values of correlator outputs, the phase estimates are determined as

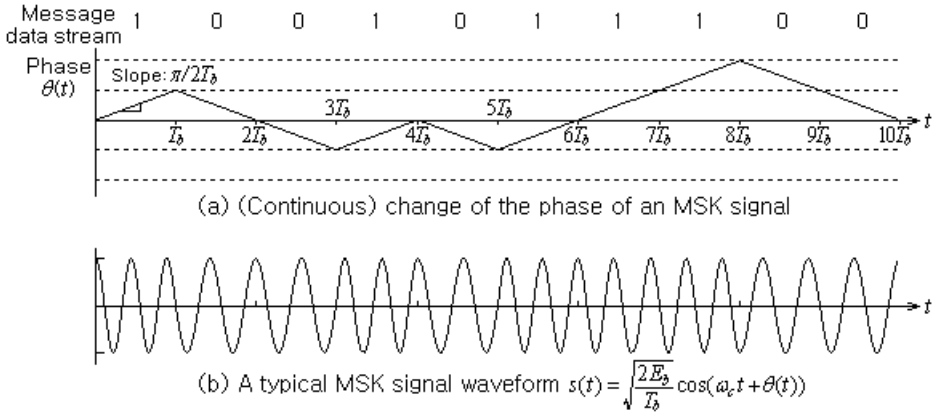
$$\theta(2kT_b) = \begin{cases} 0 & \text{if } y_c((2k+1)T_b) > 0 \\ \pm\pi \text{ (close to } \theta((2k-1)T_b)) & \text{if } y_c((2k+1)T_b) < 0 \end{cases} \qquad \text{(P7.6.10a)}$$

$$\theta((2k-1)T_b) = \begin{cases} -\pi/2 \text{ if } y_s(2kT_b) > 0 \\ +\pi/2 \text{ if } y_s(2kT_b) < 0 \end{cases} \qquad \text{(P7.6.10b)}$$

so that the maximum difference between the phase estimates at successive sampling instants is $\pm\pi/2$. Then the detector judges the value of transmitted data bit to be 1 or 0 depending on whether θ turns out to have increased or decreased.

Noting that the sampled values of correlator outputs $y_c((2k+1)T_b)$ and $y_s(2kT_b)$ will be distributed around $(+\sqrt{E_b}, 0)$ and $(-\sqrt{E_b}, 0)$ in the signal space and thus the (minimum) distance between signal points is $d_{\min} = 2\sqrt{E_b}$, the (symbol or bit) error probability is

$$P_{e,b} \overset{(5.2.35)}{=} Q\left(\frac{d_{\min}/2}{\sigma=\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{E_b}{N_0/2}}\right) = Q\left(\sqrt{SNR_{r,b}}\right) \qquad \text{(P7.6.11)}$$



(a) (Continuous) change of the phase of an MSK signal

(b) A typical MSK signal waveform $s(t) = \sqrt{\frac{2E_b}{T_b}}\cos(\omega_c t + \theta(t))$

Figure P7.6 The phase and waveform of an MSK signal

```
%sim_MSK.m
% simulates a digital communication system with MSK signaling
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
b=1; M=2^b; % # of bits per symbol and modulation order
SNRbdBt=0:0.1:12;   SNRbt=10.^(SNRbdBt/10);
pbe_PSK= prob_error(SNRbdBt,'PSK',b,'bit');
pbe_FSK= prob_error(SNRbdBt,'FSK',b,'bit');
Tb=1; Ts=b*Tb; % Bit/Symbol time
Nb=16; Ns=Nb*b; % # of sample times in Tb and Ts
T=Ts/Ns; LB=4*Ns; LBN2=LB-2*Ns+1; % Sample time and Buffer size
Es=2;  % Energy of signal waveform
% QPSK signal waveforms
ds=[0 1];
wc=2*pi; wcT=wc*T;
t=[0:2*Nb-1]*T; pihTbt=pi/2/Tb*t;
su=sqrt(2/Tb)*[cos(pihTbt-pi/2).*cos(wc*(t-Tb));
               ??????????????????????]; % Eq.(P7.6.8)
su=[fftshift(su(1,:)); su(2,:)];  suT=su*T;
ik=[1 2 1 2 1 1 2 2 2 1];
sq2EbTb=sqrt(2*Es/b/Tb); pihTbT=pi/2/Tb*T;
SNRdBs=[1 3 10]; % Range of SNRbdB
MaxIter=10000; % Number of iterations
for iter=1:length(SNRdBs)
   SNRbdB=SNRdBs(iter);  SNR=10^(SNRbdB/10);
   N0=2*(Es/b)/SNR; sigma2=N0/2; sgmsT=sqrt(sigma2/T);
   yr=zeros(2,LB); yc=0; ys=0; th0=0; t=0; wct=0; tht=th0; mn=1;
   nobe= 0; % Number of bit errors to be accumulated
   for k=1:MaxIter
     i=ceil(rand*M); s(k)=ds(i); sgn=s(k)*2-1;
     for m=1:Nb % Operation per bit time
        bp_noise= randn*cos(wct)-randn*sin(wct);
        rn= sq2EbTb*cos(wct+tht) + sgmsT*bp_noise;
        yr=[yr(:,2:LB) suT(:,mn)*rn]; % Correlator output - DTR input
        t=t+T; wct=wct+wcT; tht=tht+sgn*pihTbT; mn=mn+1;
     end
     if tht<-pi2, tht=tht+pi2; elseif tht>pi2, tht=tht-pi2; end
     thtk(k+1)=tht;
     if mn>size(su,2), mn=1; end
     if mod(k,2)==1
        yc=sum(yr(?,LBN2:LB));
        th_hat(k)=??*(yc<0);
        % if k>1&th_hat(k-1)<0, th_hat(k)=-th_hat(k); end
      else
        ys=sum(yr(?,LBN2:LB));
        th_hat(k)=????*(2*(ys<0)-1);
     end
     if k>1 % Detector(DTR)
       d=(dth_hat>0);  if abs(dth_hat)>=pi, d=~d; end
       nobe = nobe+(d~=s(???)); if nobe>100; break; end
     end
   end
   pobe(iter)= nobe/((k-1)*b);
end
semilogy(SNRbdBt,pbe_PSK,'k', SNRbdBt,pbe_FSK,'k:', SNRdBs,pobe,'b*')
```

Compared with Eq. (7.2.9), which is the error probability for BFSK signaling, the SNR has increased by two times, which is attributed to the doubled integration period $2T_b$ of the correlators.

(a) Complete the above incomplete program "sim_MSK.m" so that it can simulate the MSK modulation and demodulation process.

(b) Consider the in-phase/quadrature symbol shaping functions

$$g_c(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{\pi}{2T_b}t\right) & \text{for } -T_b \le t \le T_b \\ 0 & \text{elsewhere} \end{cases} \tag{P7.6.12a}$$

$$g_s(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi}{2T_b}t\right) & \text{for } 0 \le t \le 2T_b \\ 0 & \text{elsewhere} \end{cases} \tag{P7.6.12b}$$

Verify that their CTFTs and ESDs (energy spectral densities) can be obtained as

$$\begin{aligned} G_c(\omega) = \mathcal{F}\{g_c(t)\} &\overset{\text{(1.3.2a)}}{\underset{\text{(P7.6.12a)}}{=}} \sqrt{\frac{2E_b}{T_b}} \int_{-T_b}^{T_b} \cos\left(\frac{\pi}{2T_b}t\right) e^{-j\omega t}\, dt \\ &= \sqrt{\frac{E_b}{2T_b}} \int_{-T_b}^{T_b} \left(e^{j\pi t/2T_b} + e^{-j\pi t/2T_b}\right) e^{-j\omega t}\, dt \\ &= \sqrt{\frac{E_b}{2T_b}} \left(\frac{e^{j(\pi/2T_b-\omega)t}}{j(\pi/2T_b-\omega)} - \frac{e^{-j(\pi/2T_b+\omega)t}}{j(\pi/2T_b+\omega)} \right)\Bigg|_{-T_b}^{T_b} \\ &= \sqrt{\frac{E_b}{2T_b}} \left(\frac{2\sin(\pi/2T_b-\omega)T_b}{\pi/2T_b-\omega} + \frac{2\sin(\pi/2T_b+\omega)T_b}{\pi/2T_b+\omega} \right) \\ &= \sqrt{\frac{2E_b}{T_b}} \frac{(\pi/T_b)\sin(\pi/2)\cos\omega T_b - 2\omega\cos(\pi/2)\sin\omega T_b}{(\pi/2T_b)^2-\omega^2} \\ &= \sqrt{\frac{2E_b}{T_b}} \frac{(\pi/T_b)\cos\omega T_b}{(\pi/2T_b)^2-\omega^2} \end{aligned} \tag{P7.6.13a}$$

$$\begin{aligned} G_s(\omega) = \mathcal{F}\{g_s(t)\} &\overset{\text{(1.3.2a)}}{\underset{\text{(P7.6.12b)}}{=}} \sqrt{\frac{2E_b}{T_b}} \int_0^{2T_b} \sin\left(\frac{\pi}{2T_b}t\right) e^{-j\omega t}\, dt \\ &= \sqrt{\frac{2E_b}{T_b}} \int_{-T_b}^{T_b} \sin\left(\frac{\pi}{2T_b}(t+T_b)\right) e^{-j\omega(t+T_b)}\, dt \\ &= \sqrt{\frac{2E_b}{T_b}} \int_{-T_b}^{T_b} \cos\left(\frac{\pi}{2T_b}\right) e^{-j\omega t}\, dt\; e^{-j\omega T_b} \\ &\overset{\text{(P7.6.13a)}}{=} \sqrt{\frac{2E_b}{T_b}} \frac{(\pi/T_b)\cos\omega T_b}{(\pi/2T_b)^2-\omega^2} e^{-j\omega T_b} \end{aligned} \tag{P7.6.13b}$$

$$;\ \Phi_{gc}(\omega) = \Phi_{gs}(\omega) = \left|G_{gc}(\omega)\right|^2 = \left|G_{gs}(\omega)\right|^2 = \frac{2\pi^2 E_b}{T_b^3} \frac{\cos^2 \omega T_b}{((\pi/2T_b)^2-\omega^2)^2} \tag{P7.6.14}$$

7.7 Simulation of FSK Using Simulink

Fig. P7.7.1 shows a Simulink model ("FSK_passband_sim.mdl") to simulate a BFSK (binary frequency shift keying) passband communication system and Fig. P7.7.2 shows the subsystem for the demodulator and detector where the following parameters are to be set in the workspace:

```
b=1; M=2^b;              % Number of bits per symbol and Modulation order
Ns=40; Ts=1e-5; T=Ts/Ns; % Symbol time and Sample time
dw=2*pi/Ts; wc=10*dw;    % Frequency spacing and Carrier freq[rad/s]
EbN0dB=10                % Eb/N0[dB]
Target_no_of_error=50;   % Simulation stopping criterion
```

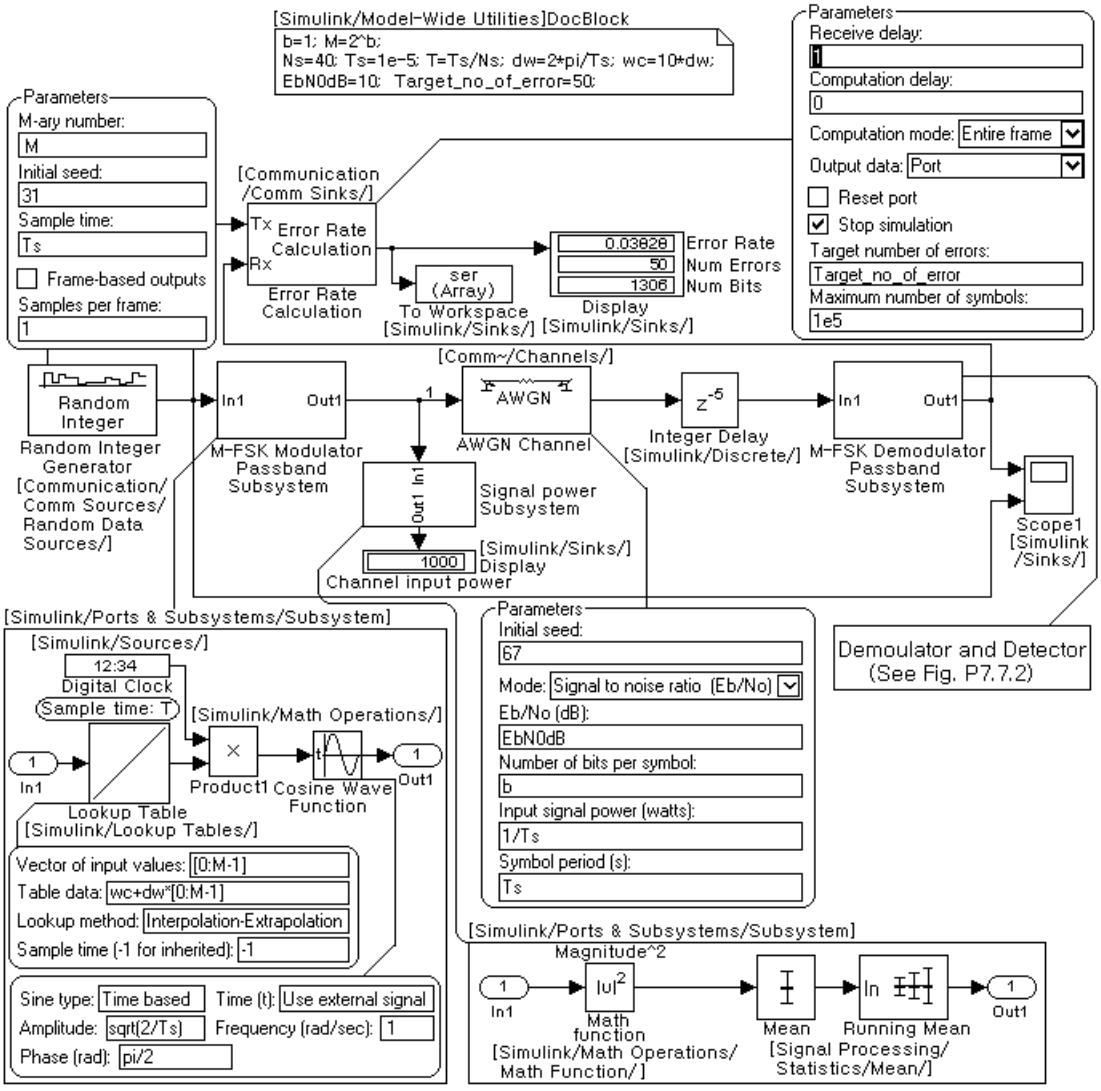


Figure P7.7.1 Simulink model for simulating a noncoherent BFSK communication system ("FSK_passband_sim.mdl")

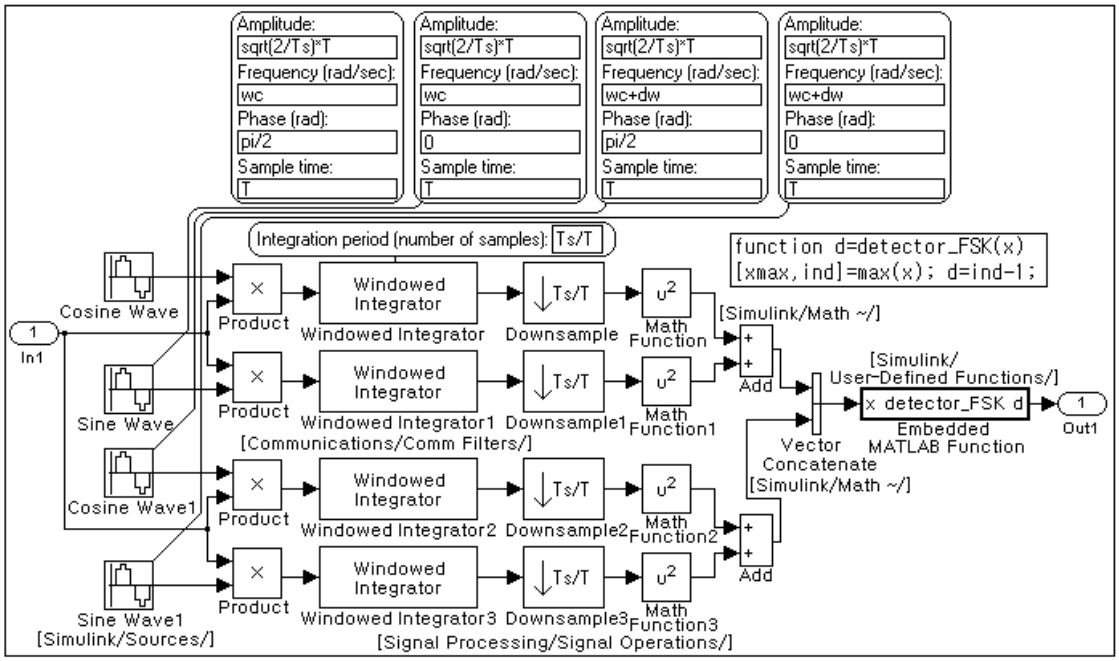


Figure P7.7.2 The subsystem for the demodulator and detector

```
%do_FSK_sim.m
clear, clf
b=1; M=2^b; % Number of bits per symbol and Modulation order
Nbsps=b*2^3;  % # of samples per symbol in baseband
Nos=5; Ns=Nbsps*Nos; % # of samples per symbol in passband
Ts=1e-5; T=Ts/Ns; % Symbol time and Sample time
dw=2*pi/Ts; % Frequency separation [rad/s]
wc=10*dw; % Carrier Frequency[rad/s] (such that wc*T<pi)
Target_no_of_error=50; SNRdBs=[5  10]; EbN0dBs=SNRdBs-3;
for i=1:length(SNRdBs)
   SNRdB=SNRdBs(i); EbN0dB=SNRdB-3; % Eb/(N0/2)=SNR-> Eb/N0=SNR/2
   sim('FSK_passband_sim',1e5*Ts); SERs(i)=ser(end,1);
end
SNRdBt=0:0.1:13;
poset = prob_error(SNRdBt,'FSK',b,'sym','noncoherent');
semilogy(SNRdBt,poset,'k', SNRdBs,SERs,'*'), xlabel('SNR[dB]')
Transmitted_Signal_Power = 1/Ts;
Received_Signal_Power = ...
  Transmitted_Signal_Power*(1+10^(-(EbN0dBs(end)+3)/10)*Ts/b/T)
```

```
function d=detector_FSK(z2s)
[z2max,ind]=max(z2s); d=ind-1; % Fig. 7.4.2
```

Once the Simulink model named “FSK_passband_sim.mdl” is opened, it can be run by clicking on the Run button on the tool bar or by using the MATLAB command ‘sim( )’ as in the above program “do_FSK_sim.m”. You can disconnect the Display Block for signal power measurement to increase the running speed.

(cf) Note that the receive delay in the Error Rate Calculation block is set to 1 due to the sample difference between the transmitted data and detected data observed through the Scope.

(a) After having composed and saved the Simulink model "FSK_passband_sim.mdl" (with no delay) and two MATLAB programs "do_FSK_sim.m" and "detector_FSK.m" (that can be substituted by an Embedded MATLAB Function Block), run the MATLAB program "do_FSK_sim.m" to get the SERs (SER: symbol error rate) for a BFSK communication system with SNRdBs=[5 10]. Does the simulation result conform with that in Fig. 7.5 (for $b=1$) or 7.6.2?

(b) Change the 7th statement of the MATLAB program "do_FSK_sim.m" into 'dw=pi/Ts' or 'dw=3*pi/Ts' and run the program to get the SER. How are the SERs compared with those obtained in (a)? What does the difference come from?

(c) Insert the delay of 1~5 samples and discuss the sensitivity of the system w.r.t. such a delay in terms of the SER performance in connection with coherence or noncoherence of the communication system.

(d) Referring to Fig. 7.4.2, extend the Simulink model into "FSK4_passband_sim.mdl" so that it can simulate an $M=2^2=4$-ary FSK communication system and run it to get the SER curve.

(e) To understand how large the power of noise added by the AWGN Block is, connect the Signal power Subsystem together with the Display Block into the input of the AWGN block, click on the Run button, and read the displayed value of the channel input signal power. Then connect the Signal power Subsystem together with the Display Block into the output of the AWGN Block, click on the Run button, and read the displayed value of the channel output signal power that will be the sum of the input signal power and noise power (variance). Do the channel input and output powers measured through the Simulink simulation conform with those obtained using the last two statements in the above program "do_FSK_sim.m":

```
Transmitted_Signal_Power = 1/Ts;
Received_Signal_Power = ...
   Transmitted_Signal_Power*(1+10^(-(EbN0dBs(end)+3)/10)*Ts/b/T)
```

Why is 3 added to `EbN0dBs(end)` in the above statement?

(cf) Visit the webpage <http://www.mathworks.com/access/helpdesk/help/helpdesk.html> and refer to the explanation about the function of AWGN Block that can be found in the user manual for Communications Blockset, according to which the SNR is defined to be

$$SNR\text{dB} = 10\log_{10}\frac{\text{Signal Power}}{\text{Noise Variance}}$$

$$= \begin{cases} 10\log_{10}\dfrac{E_s/T_s}{N_0/T} = EsN0\text{dB} + 10\log_{10}\dfrac{T}{T_s} = EbN0\text{dB} + 10\log_{10} b\dfrac{T}{T_s} \\ \qquad\qquad \text{for a complex-valued signal} \\ 10\log_{10}\dfrac{E_s/T_s}{(N_0/2)T} = EsN0\text{dB} + 10\log_{10}\dfrac{2T}{T_s} = EbN0\text{dB} + 10\log_{10} b\dfrac{T}{T_s} + 3 \\ \qquad\qquad \text{for a real-valued signal} \end{cases} \quad \text{(P7.7.1)}$$

$$; \frac{\text{Signal Power}}{\text{Noise Variance}} = \begin{cases} (T_s/bT)10^{-EbN0dB/10} & \text{for a complex-valued signal} \\ (T_s/bT)10^{-(EbN0dB+3)/10} & \text{for a real-valued signal} \end{cases} \quad \text{(P7.7.2)}$$

7.8 Simulation of PSK Using Simulink

Fig. P7.8.1 shows a Simulink model to simulate a PSK (phase shift keying) passband communication system where the following parameters are to be set in the workspace:

```
b=1; M=2^b;              % Number of bits per symbol and Modulation order
Ns=40; Ts=1e-5; T=Ts/Ns; % Symbol time and Sample time
wc=2*pi*10/Ts;           % Carrier freq[rad/s]
EbN0dB=7;                % Eb/N0[dB]
Target_no_of_error=20;   % Simulation stopping criterion
```

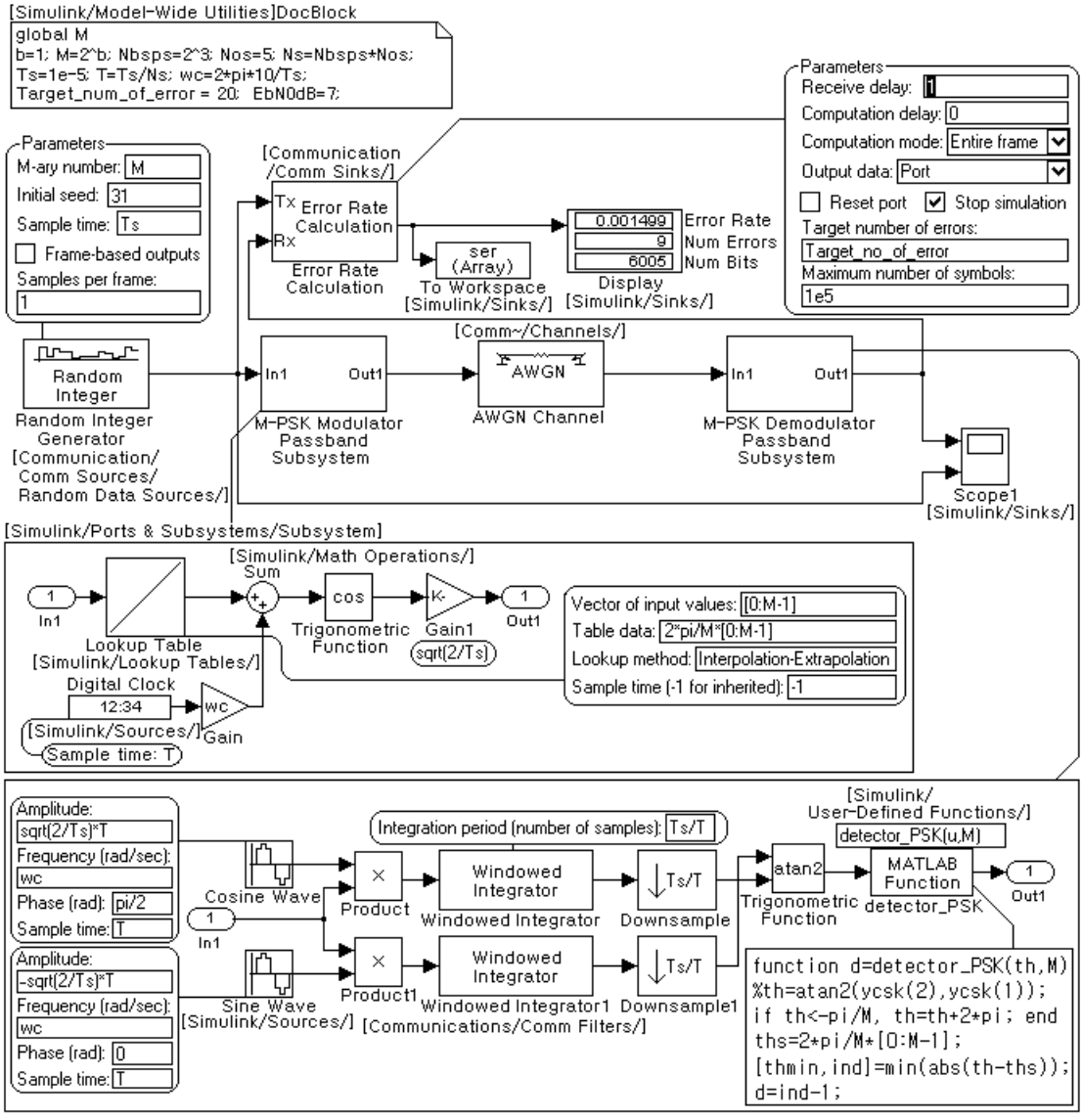


Figure P7.8.1 Simulink model for simulating a PSK communication system ("PSK_passband_sim.mdl")

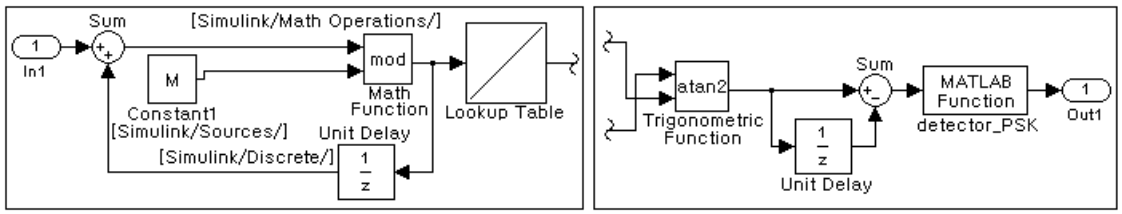
```
%do_PSK_sim.m
clear, clf
b=1; M=2^b; % Number of bits per symbol and Modulation order
Nbsps=b*2^3;  % # of samples per symbol in baseband
Nos=5; Ns=Nbsps*Nos; % # of samples per symbol in passband
Ts=1e-5; T=Ts/Ns; % Symbol time and Sample time
wc=2*pi*10/Ts; % Carrier Frequency[rad/s] (such that wc*T<pi)
Target_no_of_error=20;
SNRdBs=[5  10]; EbN0dBs=SNRdBs-3;
for i=1:length(SNRdBs)
   SNRdB=SNRdBs(i); EbN0dB=SNRdB-3; % Eb/(N0/2)=SNR-> Eb/N0=SNR/2
   sim('PSK_passband_sim',1e5*Ts);
   SERs(i) = ser(end,1);
end
SNRdBt=0:0.1:13;
poset = prob_error(SNRdBt,'PSK',b,'sym');
semilogy(SNRdBt,poset,'k', SNRdBs,SERs,'*'), xlabel('SNR[dB]')
Transmitted_Signal_Power = 1/Ts;
Received_Signal_Power = ...
   Transmitted_Signal_Power*(1+10^(-(EbN0dBs(end)+3)/10)*Ts/b/T)
```

```
function d=detector_PSK(th,M)
%th=atan2(ycsk(2),ycsk(1));
if th<-pi/M, th=th+2*pi; end
[thmin,ind]=min(abs(th-2*pi/M*[0:M-1])); % Eq.(7.3.9) or Fig. 7.7
d=ind-1;
```

(a) After having composed and saved the Simulink model "PSK_passband_sim.mdl" and two MATLAB programs "do_PSK_sim.m" and "detector_PSK.m", run the MATLAB program "do_PSK_sim.m" to get the SERs for a BPSK communication system with SNRdBs=[5 10]. Does the simulation result conform with that in Fig. 7.10 (for $b=1$)?

(b) Modify the MATLAB program "do_PSK_sim.m" (possibly together with) Simulink model "PSK_passband_sim.mdl" to simulate an $M=2^2=4$-ary PSK (QPSK) communication system and run it to get the SER curve.

(c) Rename the Simulink model "PSK_passband_sim.mdl" "DPSK_passband_sim.mdl" and modify it to simulate an $M=2^2=4$-ary DPSK (QDPSK) communication system. You can refer to the MATLAB program "sim_DPSK_passband.m" and Fig. P7.8.2. Run it to get the SERs with SNRdBs=[5 10].



(a) Extension of the modulator block (b) Extension of the detector block

Figure P7.8.2 Modification of the modulator and detector blocks for DPSK

7.9 Simulation of QAM Using Simulink

Fig. P7.9 shows a Simulink model to simulate a QAM (quadrature amplitude modulation) passband communication system where the following parameters are to be set in the workspace:

```
b=4; M=2^b; L=2^(b/2); % Number of bits per symbol and Modulation order
Ns=40; Ts=1e-5; T=Ts/Ns; % Symbol time and Sample time
wc=2*pi*10/Ts;           % Carrier freq[rad/s]
EbN0dB=10;               % Eb/N0[dB]
Target_no_of_error=100;  % Simulation stopping criterion
```

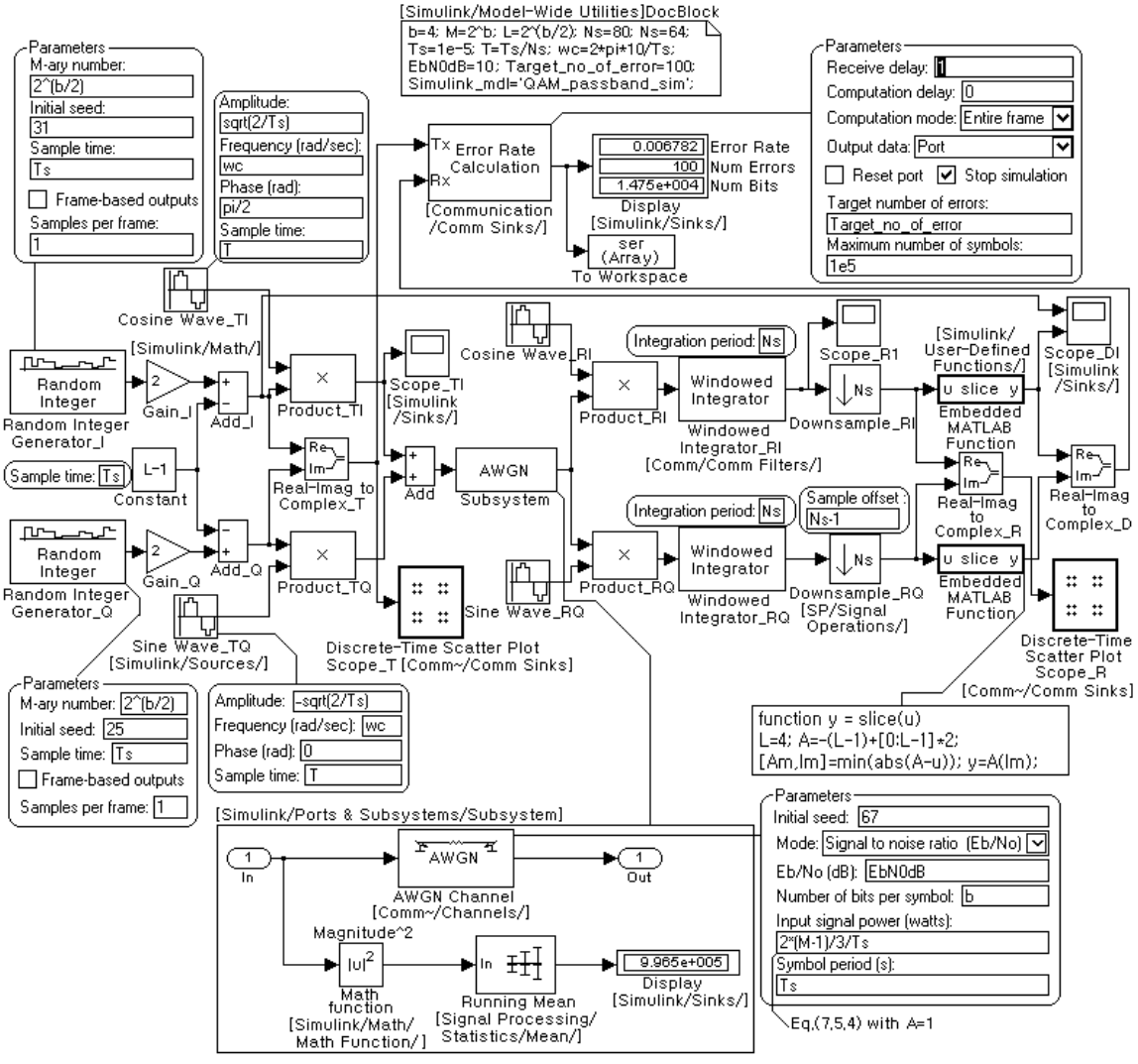


Figure P7.9 Simulink model for simulating a QAM communication system ("QAM_passband_sim.mdl")

```
%do_QAM_sim.m
clear, clf
b=4; M=2^b; L=2^(b/2); % # of bits per symbol and Modulation order
Ns=b*10; Ts=1e-5; T=Ts/Ns; % Symbol time and Sample time
wc=2*pi*10/Ts; % Carrier Frequency[rad/s] (such that wc*T<pi)
Target_no_of_error=100;
SNRdBs=[5  10]; EbN0dBs=SNRdBs-3;
for i=1:length(SNRdBs)
   SNRdB=SNRdBs(i); EbN0dB=SNRdB-3; % Eb/(N0/2)=SNR-> Eb/N0=SNR/2
   sim('QAM_passband_sim',1e5*Ts); SERs(i) = ser(end,1);
end
SNRdBt=0:0.1:13; poset = prob_error(SNRdBt,'QAM',b,'sym');
semilogy(SNRdBt,poset,'k', SNRdBs,SERs,'*'), xlabel('SNR[dB]')
Transmitted_Signal_Power = 2*(M-1)/3/Ts;
Received_Signal_Power = ...
   Transmitted_Signal_Power*(1+10^(-(EbN0dBs(end)+3)/10)*Ts/b/T)
```

```
function a=slice(x,A)
[Am,Im]=min(abs(A-x)); a=A(Im);
```

(a) After having composed and saved the Simulink model "QAM_passband_sim.mdl" and two MATLAB programs "do_QAM_sim.m" and "slice.m" (that can be substituted by an Embedded MATLAB Function Block), run the MATLAB program "do_QAM_sim.m" to get the SERs for SNRdBs=[5 10]. Does the simulation result conform with the theoretical symbol error probability that can be obtained using the MATLAB routine "prob_error()" in Sec. 7.6?

(b) Modify the program "do_QAM_sim.m" and/or Simulink model "QAM_passband_sim.mdl" to simulate an $M=2^6=64$ -ary QAM communication system and run it to get the SER curve.

```
%do_MSK_sim.m
clear, clf
SNRdBt=[0:0.1:10]; SNRdBs=[5  10]; EbN0dBs= SNRdBs-3;
b=1; M=2^b; % Number of bits per symbol and Modulation order
Tb=1; Nb=16; T=Tb/Nb; Ts=b*Tb; % Bit duration and Sample time
wc=4*pi/Tb; % Carrier frequency[rad/s]
t=[0:2*Nb-1]*T;  pihTbt=pi/2/Tb*t;
suT=sqrt(2/Tb)*T*[cos(pihTbt-pi/2).*cos(wc*(t-Tb));
                  sin(pihTbt).*sin(wc*t)];  % Eq. (7.6.8a,b)
M_filter1=fliplr(suT(1,:)); M_filter2=fliplr(suT(2,:));
Target_no_of_error = 50;
Simulink_mdl='MSK_passband_sim';
for i=1:length(SNRdBs)
   SNRdB = SNRdBs(i);  EbN0dB = SNRdB-3;
   sim(Simulink_mdl,1e5*Tb),  BERs(i) = ber(end,1);
end
pobet_PSK=prob_error(SNRdBt,'PSK',b,'bit');
pobet_FSK=prob_error(SNRdBt,'FSK',b,'bit');
semilogy(SNRdBt,pobet_PSK,'k', SNRdBt,pobet_FSK,'k:', ...
    SNRdBs,BERs,'b*')
```

```
function d=detector_MSK(dth)
if dth>0, d=1; else d=0; end
if abs(dth)>pi, d=1-d; end
```

7.10 Simulation of MSK Using Simulink

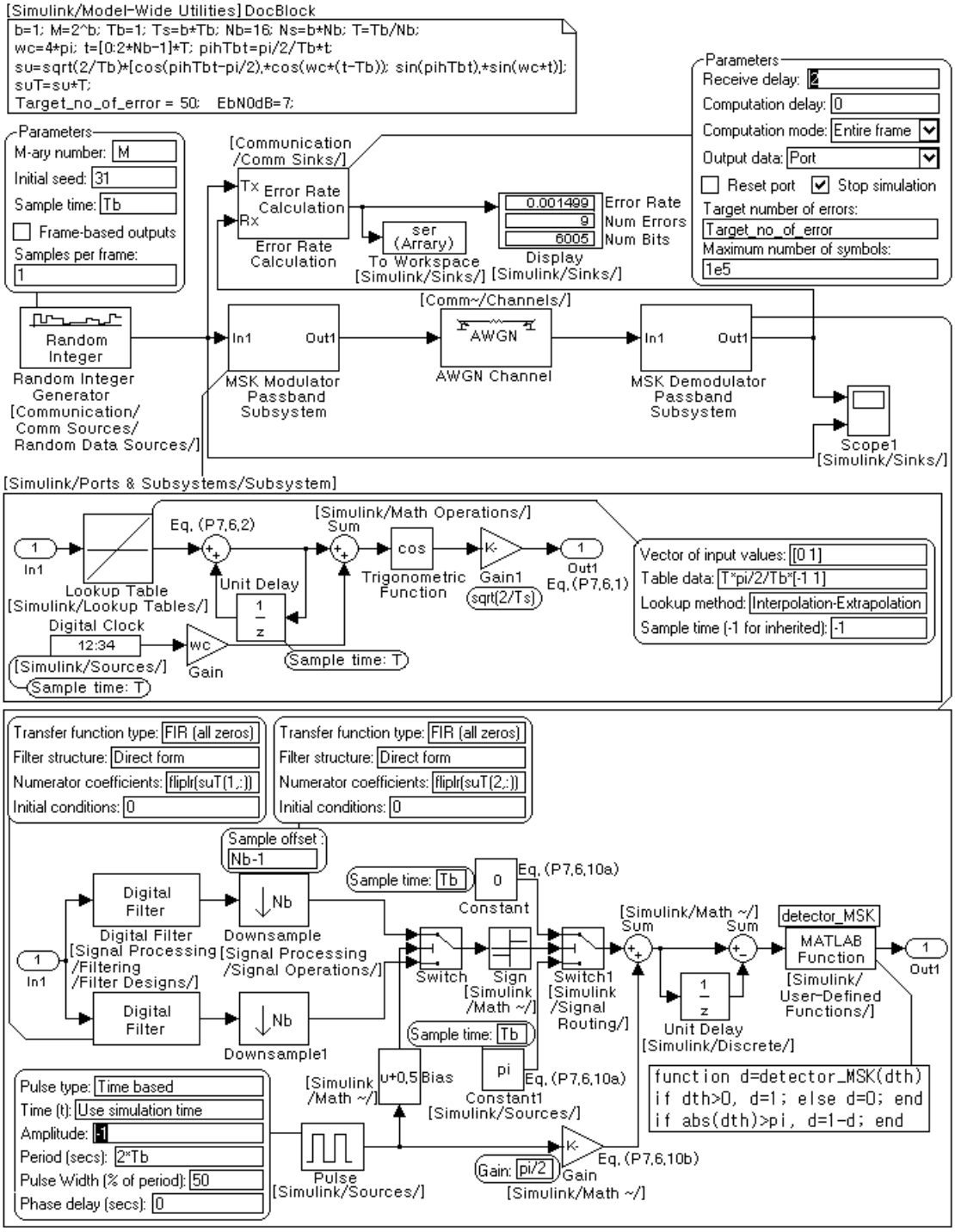


Figure P7.10 Simulink model for simulating an MSK communication system ("MSK_passband_sim.mdl")

Fig. P7.10 shows a Simulink model to simulate an MSK (minimum shift keying) passband communication system where the following parameters are to be set in the workspace. Note that the demodulator is implemented using matched (FIR) filters (whose coefficient vectors or impulse responses are the reversed and delayed versions of the basis signal waveforms (P7.6.8a,b)) instead of correlators. Having composed and saved the Simulink model "MSK_passband_sim.mdl" and two MATLAB programs "do_MSK_sim.m" and "detector_MSK.m", run the MATLAB program "do_MSK_sim.m" to get the BERs for SNRdBs=[5 10]. Does the simulation result go into between the theoretical symbol error probabilities for FSK and PSK?

```
b=1; M=2^b;          % Number of bits per symbol and Modulation order
Nb=16; Ns=b*Nb;
Tb=1e-5; Ts=b*Tb; T=Ts/Ns; % Symbol/Bit time and Sample time
wc=2*pi*10/Tb;              % Carrier freq[rad/s]
t=[0:2*Nb-1]*T;  pihTbt=pi/2/Tb*t;
suT=sqrt(2/Tb)*T*[cos(pihTbt-pi/2).*cos(wc*(t-Tb));
                  sin(pihTbt).*sin(wc*t)];
M_filter1=fliplr(suT(1,:)); M_filter2=fliplr(suT(2,:));
EbN0dB=7;                   % Eb/N0[dB]
Target_no_of_error=20;    % Simulation stopping criterion
```

9.4.3 Cyclic Coding

A *cyclic code* is a linear block code having the property that a cyclic shift (rotation) of any codeword yields another codeword. Due to this additional property, the encoding and decoding processes can be implemented more efficiently using a feedback shift register. An (N, K) cyclic code can be described by an $(N-K)^{\text{th}}$-degree generator polynomial

$$\mathbf{g}(x) = g_0 + g_1 x + g_2 x^2 + \cdots + g_{N-K} x^{N-K} \tag{9.4.24}$$

The procedure of encoding a K-bit message vector $\mathbf{m} = [m_0\ m_1 \cdots m_{K-1}]$ represented by a $(K-1)^{\text{th}}$-degree polynomial

$$\mathbf{m}(x) = m_0 + m_1 x + m_2 x^2 + \cdots + m_{K-1} x^{K-1} \tag{9.4.25}$$

into an N-bit codeword represented by an $(N-1)^{\text{th}}$-degree polynomial is as follows:

1. Divide $x^{N-K}\mathbf{m}(x)$ by the generator polynomial $\mathbf{g}(x)$ to get the remainder polynomial $\mathbf{r}_m(x)$.
2. Subtract the remainder polynomial $\mathbf{r}_m(x)$ from $x^{N-K}\mathbf{m}(x)$ to obtain a codeword polynomial

$$\begin{aligned}\mathbf{c}(x) &= x^{N-K}\mathbf{m}(x) \oplus \mathbf{r}_m(x) = \mathbf{q}(x)\mathbf{g}(x) \\ &= r_0 + r_1 x + \cdots + r_{N-K-1} x^{N-K-1} + m_0 x^{N-K} + m_1 x^{N-K+1} + \cdots + m_{K-1} x^{N-1}\end{aligned} \tag{9.4.26}$$

which has the generator polynomial $\mathbf{g}(x)$ as a (multiplying) factor. Then the first $(N-K)$ coefficients constitute the parity vector and the remaining K coefficients make the message vector. Note that all the operations involved in the polynomial multiplication, division, addition, and subtraction are not the ordinary arithmetic ones, but the modulo-2 operations.

(**Example 9.6**) A Cyclic Code

With a (7,4) cyclic code represented by the generator matrix

$$\mathbf{g}(x) = g_0 + g_1 x + g_2 x^2 + g_3 x^3 = 1 + 1 \cdot x + 0 \cdot x^2 + 1 \cdot x^3 \tag{E9.6.1}$$

find the codeword for a message vector $\mathbf{m} = [1\ 0\ 1\ 1]$.

Noting that $N = 7,\ K = 4$, and $N - K = 3$, we divide $x^{N-K}\mathbf{m}(x)$ by $\mathbf{g}(x)$ as

$$\begin{aligned}x^{N-K}\mathbf{m}(x) &= x^3(1 + 0 \cdot x + 1 \cdot x^2 + 1 \cdot x^3) = x^6 + x^5 + x^3 = \mathbf{q}(x)\mathbf{g}(x) + \mathbf{r}_m(x) \\ &= (x^3 + x^2 + x + 1)(x^3 + x + 1) + 1 \ \text{(modulo-2 operation)}\end{aligned} \tag{E9.6.2}$$

to get the remainder polynomial $\mathbf{r}_m(x) = 1$ and add it to $x^{N-K}\mathbf{m}(x) = x^3\mathbf{m}(x)$ to make the codeword polynomial as

$$\mathbf{c}(x) = \mathbf{r}_m(x) + x^3\mathbf{m}(x) = 1 + 0 \cdot x + 0 \cdot x^2 + 1 \cdot x^3 + 0 \cdot x^4 + 1 \cdot x^5 + 1 \cdot x^6 \tag{E9.6.3}$$

$$\rightarrow\ \mathbf{c} = [\underbrace{1\ \ 0\ \ 0}_{\text{parity}}\ |\ \underbrace{1\ \ 0\ \ 1\ \ 1}_{\text{message}}]$$

The codeword made in this way has the $N-K=3$ parity bits and $K=4$ message bits.

Now, let us consider the procedure of decoding a cyclic coded vector. Suppose the RCVR has received a possibly corrupted code vector $\mathbf{r}=\mathbf{c}+\mathbf{e}$ where $\mathbf{c}$ is a codeword and $\mathbf{e}$ is an error. Just as in the encoder, this received vector, being regarded as a polynomial, is divided by the generator polynomial $\mathbf{g}(x)$

$$\mathbf{r}(x) = \mathbf{c}(x)+\mathbf{e}(x) = \mathbf{q}(x)\mathbf{g}(x)+\mathbf{e}(x) = \mathbf{q}'(x)\mathbf{g}(x)+\mathbf{s}(x) \tag{9.4.27}$$

to yield the remainder polynomial $\mathbf{s}(x)$. This remainder polynomial $\mathbf{s}$ may not be the same as the error vector $\mathbf{e}$, but at least it is supposed to have a crucial information about $\mathbf{e}$ and therefore, may well be called the *syndrome*. The RCVR will find the error pattern $\mathbf{e}$ corresponding to the syndrome $\mathbf{s}$, subtract it from the received vector $\mathbf{r}$ to get hopefully the correct codeword

$$\mathbf{c} = \mathbf{r} \oplus \mathbf{e} \tag{9.4.28}$$

and accept only the last K bits (in ascending order) of this corrected codeword as a message.

The polynomial operations involved in encoding/decoding every block of message/coded sequence seem to be an unbearable computational load. However, we fortunately have divider circuits which can perform such a modulo-2 polynomial operation. Fig. 9.9 illustrates the two divider circuits (consisting of linear feedback shift registers) each of which carries out the modulo-2 polynomial operations for encoding/decoding with the cyclic code given in Example 9.6. Note that the encoder/decoder circuits process the data sequences in descending order of polynomial.

The encoder/decoder circuits are cast into the MATLAB routines 'cyclic_encoder()' and 'cyclic_decoder0()', respectively, and we make a program "do_cyclic_code.m" that uses the two routines 'cyclic_encoder()' and 'cyclic_decoder()' (including 'cyclic_encoder0()') to simulate the encoding/decoding process with the cyclic code given in Example 9.6. Note a couple of things about the decoding routine 'cyclic_decoder()':

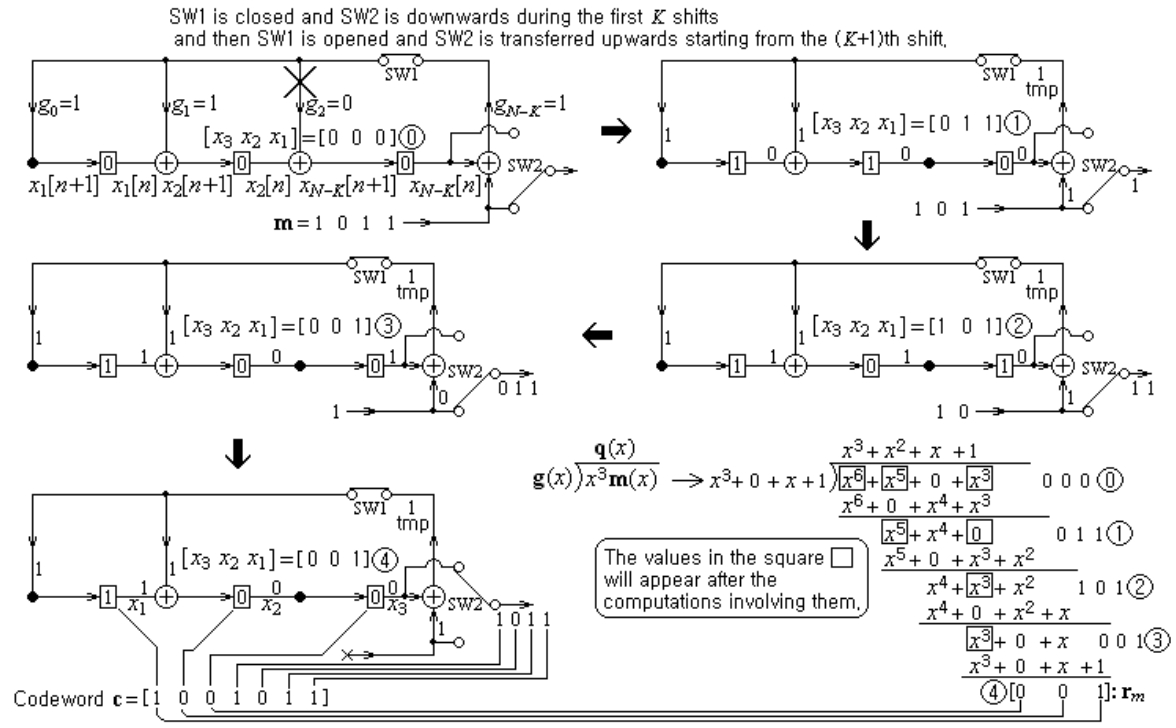
- It uses a table of error patterns in the matrix E, which has every correctable error pattern in its rows. The table is searched for a suitable error pattern by using an error pattern index vector epi, which is arranged by the decimal-coded syndrome and therefore, can be addressed efficiently by a syndrome just like a decoding hardware circuit.
- If the error pattern table E and error pattern index vector epi are not supplied from the calling program, it uses 'cyclic_decoder0()' to supply itself with them.

```
% do_cyclic_code.m
% tries with a cyclic code.
clear
N=7; K=4; % N=15; K=7; % Codeword (Block) length and Message size
%N=31; K=16;
g=cyclpoly(N,K); g_=fliplr(g);
lm=5*K; msg= randint(1,lm);
% N=7; K=4; g_=[1 1 0 1]; g=fliplr(g_); msg=[1 0 1 1];
coded = cyclic_encoder(msg,N,K,g_); lc=length(coded);
no_transmitted_bit_errors=ceil(lc*0.05);
errors=randerr(1,lc,no_transmitted_bit_errors);
r = rem(coded+errors,2); % Received sequence
decoded = cyclic_decoder(r,N,K,g_); nobe=sum(decoded~=msg)
coded1 = encode(msg,N,K,'cyclic',g); % Use the Communication Toolbox
r1 = rem(coded1+errors,2); % Received sequence
decoded1 = decode(r1,N,K,'cyclic',g); nobe1=sum(decoded1~=msg)
```

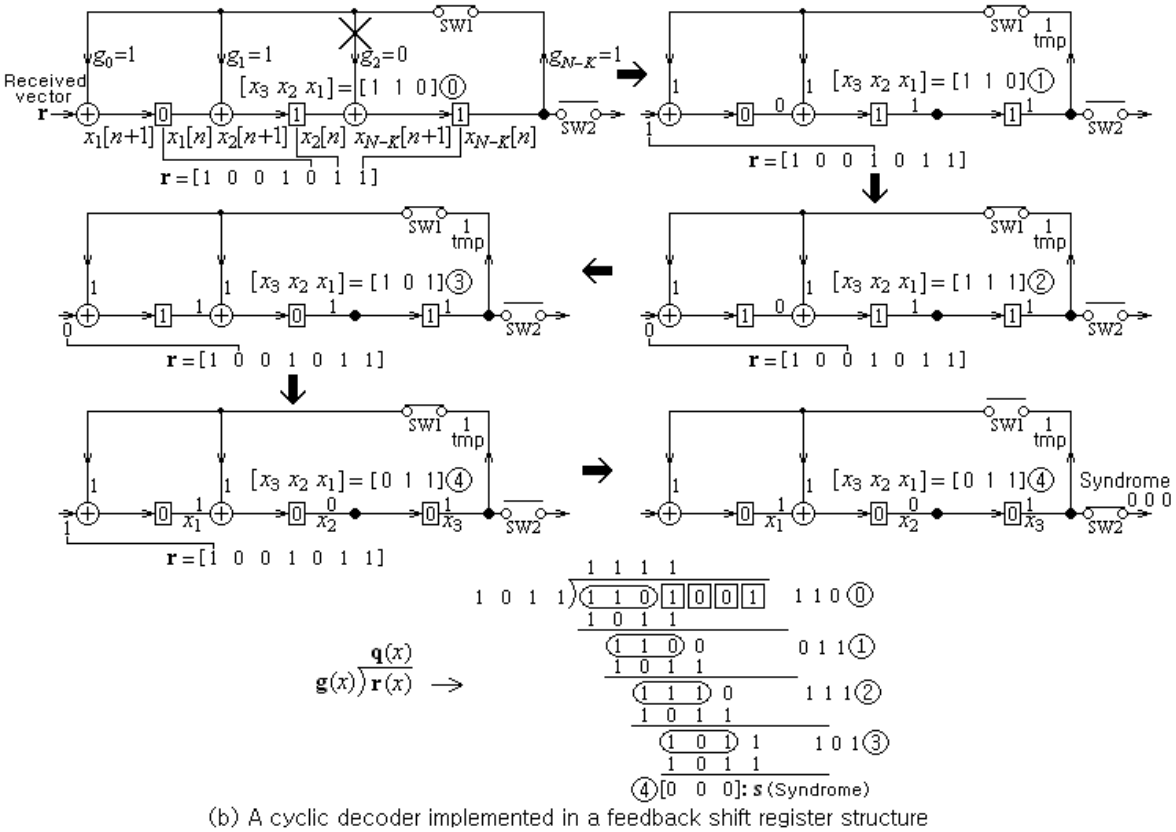
```
function coded= cyclic_encoder(msg_seq,N,K,g)
% Cyclic (N,K) encoding of input msg_seq m with generator polynomial g
Lmsg=length(msg_seq); Nmsg=ceil(Lmsg/K);
Msg= [msg_seq(:); zeros(Nmsg*K-Lmsg,1)];
Msg= reshape(Msg,K,Nmsg).';
coded= [];
for n=1:Nmsg
   msg= Msg(n,:);
   for i=1:N-K, x(i)=0; end
   for k=1:K
      tmp= rem(msg(K+1-k)+x(N-K),2); % msg(K+1-k)+g(N-K+1)*x(N-K)
      for i=N-K:-1:2,  x(i)= rem(x(i-1)+g(i)*tmp,2);  end
      x(1)=g(1)*tmp;
   end
   coded= [coded x msg]; % Eq.(9.4.26)
end
```

```
function [decodes,E,epi]=cyclic_decoder(code_seq,N,K,g,E,epi)
% Cyclic (N,K) decoding of received code_seq with generator polynml g
% E:   Error Pattern matrix or syndromes
% epi: error pattern index vector
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
if nargin<6
  nceb=ceil((N-K)/log2(N+1)); % Number of correctable error bits
  E = combis(N,nceb);  % All error patterns
  for i=1:size(E,1)
     syndrome=cyclic_decoder0(E(i,:),N,K,g);
     synd_decimal=bin2deci(syndrome);
     epi(synd_decimal)=i; % Error pattern indices
  end
end
if (size(code_seq,2)==1)  code_seq=code_seq.';  end
Lcode= length(code_seq);  Ncode= ceil(Lcode/N);
Code_seq= [code_seq(:); zeros(Ncode*N-Lcode,1)];
Code_seq= reshape(Code_seq,N,Ncode).';
decodes=[]; syndromes=[];
for n=1:Ncode
  code= Code_seq(n,:);
  syndrome= cyclic_decoder0(code,N,K,g);
  si= bin2deci(syndrome);  % Syndrome index
  if 0<si&si<=length(epi)  % Syndrome index to error pattern index
    m=epi(si);  if m>0, code=rem(code+E(m,:),2); end  % Eq.(9.4.28)
  end
  decodes=[decodes code(N-K+1:N)]; syndromes=[syndromes syndrome];
end
if nargout==2, E=syndromes; end
```

```
function x=cyclic_decoder0(r,N,K,g)
% Cyclic (N,K) decoding of an N-bit code r with generator polynomial g
for i=1:N-K, x(i)=r(i+K); end
for n=1:K
   tmp=x(N-K);
   for i=N-K:-1:2,  x(i)=rem(x(i-1)+g(i)*tmp,2);   end
   x(1)=rem(g(1)*tmp+r(K+1-n),2);
end
```



(a) A cyclic encoder implemented in a feedback shift register structure



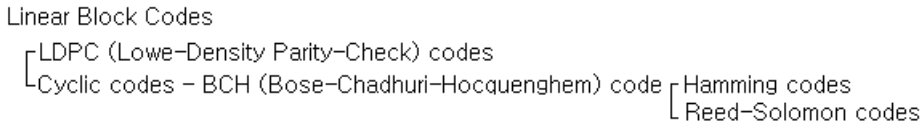
(b) A cyclic decoder implemented in a feedback shift register structure

Figure 9.9 An example of cyclic encoding/decoding processes using dividing circuits for polynomial division

Table 9.1 Communication Toolbox functions for block coding

Block coding	Related Communication Toolbox functions and objects
Linear block	encode, decode, gen2par, syndtable
Cyclic	encode, decode, cyclpoly, cyclgen, gen2par, syndtable
BCH (Bose-Chaudhuri-Hocquenghem)	bchenc, bchdec, bchgenpoly
LDPC (Low-Density Parity Check)	fec.ldpcenc, fec.ldpcdec
Hamming	encode, decode, hammgen, gen2par, syndtable
Reed-Solomon	rsenc, rsdec, rsgenpoly, rsencof, rsdecof

Table 9.1 shows a list of several Communication Toolbox functions that can be used to simulate various block coding techniques where the block codings are classfied as follows (see the MATLAB Help on Communication Toolbox - block coding):



Note the following about the functions 'encode' and 'decode' (use MATLAB Help for details):

- They can be used for any linear block coding by putting a string 'linear' and a $K \times N$ generator matrix as the fourth and fifth input arguments, respectively.
- They can be used for Hamming coding by putting a string 'hamming' as the fourth input argument or by providing them with only the first three input arguments.

The following example illustrates the usages of 'encode()'/'decode()' for cyclic coding and 'rsenc()' and 'rsdec()' for Reed-Solomon coding. Note that the *RS* (*Reed-Solomon*) *codes* are nonbinary BCH codes, which has the largest possible minimum distance for any linear code with the same message size K and codeword length N, yielding the error correcting capability of $d_c = (N-K)/2$.

```
%test_encode_decode.m  to try using encode()/decode()
N=7; K=4; % Codeword (Block) length and Message size
g=cyclpoly(N,K); % Generator polynomial for a cyclic (N,K) code
Nm=10; % # of K-bit message vectors
msg=randint(Nm,K); % Nm x K message matrix
coded = encode(msg,N,K,'cyclic',g); % Encoding
% Add bit errors with transmitted BER potbe=0.1
potbe=0.1; received=rem(coded+randerr(Nm,N,[0 1;1-potbe potbe]),2);
decoded=decode(received,N,K,'cyclic',g); % Decoding
% Probability of message bit errors after decoding/correction
pobe=sum(sum(decoded~=msg))/(Nm*K) % BER
% Usage of rsenc()/rsdec()
M=3; % Galois Field integer corresponding to the # of bits per symbol
N=2^M-1; K=3; dc=(N-K)/2; % Codeword length and Message size
msg=gf(randint(Nm,K,2^M),M); % Nm x K GF(2^M) Galois Field msg matrix
coded = rsenc(msg,N,K); % Encoding
noise = randerr(Nm,N,[1 dc+1]).*randint(Nm,N,2^M);
received = coded+noise; % Add a noise
[decoded,numerr]=rsdec(received,N,K); % Decoding
[msg decoded], numerr, pose=sum(sum(decoded~=msg))/(Nm*K) % SER
```

9.4.4 Convolutional Coding and Viterbi Decoding

In the previous sections, we discussed the block coding that encodes every K-bit block of message sequence independently of the previous message block (vector). In this section, we are going to see the *convolutional coding* that converts a K-bit message vector into an N-bit channel input sequence dependently of the previous $(L-1)K$-bit message vector (L: *constraint length*). The convolutional encoder has a structure of finite-state machine whose output depends on not only the input but also the state.

Fig. 9.10 shows a binary convolutional encoder with a $K(=2)$-bit input, an $N(=3)$-bit output, and $L-1(=3)$ 2-bit registers that can be described as a finite-state machine having $2^{(L-1)K}=2^{3\cdot 2}=64$ states. This encoder shifts the previous contents of every stage register except the right-most one into its righthand one and receives a new K-bit input to load the left-most register at an iteration, sending an N-bit output to the channel for transmission where the values of the output bits depends on the previous inputs stored in the $L-1$ registers as well as the current input.

A binary convolutional code is also characterized by N generator sequences $\mathbf{g}_1, \mathbf{g}_2, \cdots, \mathbf{g}_N$ each of which has a length of LK. For example, the convolutional code with the encoder depicted in Fig. 9.10 is represented by the $N(=3)$ generator sequences

$$\begin{aligned}\mathbf{g}_1&=[0\ \ 0\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1]\\ \mathbf{g}_2&=[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1]\\ \mathbf{g}_3&=[1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1]\end{aligned} \tag{9.4.29a}$$

which constitutes the generator (polynomial) matrix

$$G_{N\times LK}=\begin{bmatrix}\mathbf{g}_1\\ \mathbf{g}_2\\ \mathbf{g}_3\end{bmatrix}=\begin{bmatrix}0&0&1&0&1&0&0&1\\ 0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&1\end{bmatrix} \tag{9.4.29b}$$

where the value of the j^{th} element of $\mathbf{g}_i$ is 1 or 0 depending on whether the j^{th} one of the LK bits of the shift register is connected to the i^{th} output combiner or not. The shift register is initialized to all-zero state before the first bit of an input (message) sequence enters the encoder and also finalized to all-zero state by the $(L-1)K$ zero-bits padded onto the tail part of each input sequence. Besides, the length of each input sequence (to be processed at a time) is made to be MK (an integer M times K) even by zero-padding if necessary. For the input sequence made in this way so that its total length is $(M+L-1)K$ including the zeros padded onto it, the length of the output sequence is $(M+L-1)N$ and consequently, the *code rate* will be

$$R_c=\frac{MK}{(M+L-1)N}\overset{M\to\infty}{\to}\frac{K}{N}\quad\text{for } M\succ L \tag{9.4.30}$$

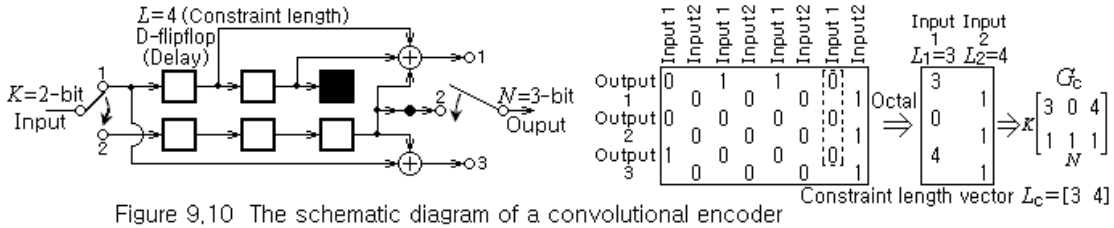


Figure 9.10 The schematic diagram of a convolutional encoder and its representation in the MATLAB functions convenc()/vitdec()

```
function [output,state]=conv_encoder(G,K,input,state,termmode)
% generates the output sequence of a binary convolutional encoder
% G     : N x LK Generator matrix of a convolutional code
% K     : Number of input bits entering the encoder at each clock cycle.
% input: Binary input sequence
% state: State of the convolutional encoder
% termmode='trunc' for no termination with all-0 state
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
if isempty(G), output=input; return; end
tmp= rem(length(input),K);
input=[input zeros(1,(K-tmp)*(tmp>0))];
[N,LK]=size(G);
if rem(LK,K)>0
  error('The number of column of G must be a multiple of K!')
end
%L=LK/K;

if nargin<4|(nargin<5 & isnumeric(state))
  input= [input zeros(1,LK)]; %input= [input zeros(1,LK-K)]; end
end
if nargin<4|~isnumeric(state)
  state=zeros(1,LK-K);
end
input_length= length(input);
N_msgsymbol= input_length/K;
input1= reshape(input,K,N_msgsymbol);
output=[];
for l=1:N_msgsymbol % Convolution output=G*input
   ub= input1(:,l).';
   [state,yb]= state_eq(state,ub,G);
   output= [output yb];
end
```

```
function [nxb,yb]=state_eq(xb,u,G)
% To be used as a subroutine for conv_encoder()
K=length(u); LK=size(G,2); L1K=LK-K;
if isempty(xb), xb=zeros(1,L1K);
 else
   N=length(xb); %(L-1)K
   if L1K~=N, error('Incompatible Dimension in state_eq()'); end
end
A=[zeros(K,L1K); eye(L1K-K) zeros(L1K-K,K)];
B=[eye(K); zeros(L1K-K,K)];
C=G(:,K+1:end); D=G(:,1:K);
nxb=rem(A*xb'+B*u',2)';
yb=rem(C*xb'+D*u',2)';
```

Given a generator matrix $G_{N\times LK}$ together with the number K of input bits and a message sequence $\mathbf{m}$, the above MATLAB routine 'conv_encoder()' pads the input sequence $\mathbf{m}$ with zeros as needed and then generates the output sequence of the convolutional encoder. Communication Toolbox has a convolutional encoding function 'convenc()' and its usage will be explained together with that of a convolutional decoding function 'vitdec()' at the end of this section.

<**Various Representations of a Convolutional Code**>

There are many ways of representing a convolutional code such as the schematic diagram like Fig. 9.10, the generator matrix like Eq. (9.4.29b), the finite-state machine (state transition diagram), the transfer function, and the trellis diagram. Fig. 9.11 illustrates the various equivalent representations of a simple convolutional code with K=1, N=2, and L=3. Especially, Fig. 9.11(c1) shows the state diagram that has 4 states corresponding to all possible contents {a=00, b=01, c=10, d=11} of the lefthand $(L-1)K$ registers in the schematic diagram (Fig. 9.11(a)) where solid/dashed branches between two states represent the state transitions in the arrow direction in response to input 0/1, respectively. Fig. 9.11(c2) is an augmented state diagram where the all-zero state a in the original state diagram (Fig. 9.11(c1)) is duplicated to make two all-zero states, one (a) with only out-ward branches and the other (a') with only in-ward branches. Note that the self-loop at the all-zero state is neglected so that the gains of all the paths starting from the all-zero state and coming back to the all-zero state can be obtained in a systematic way as follows. If we assign the gain $D^{\alpha}I^{\beta}J$ (α: the Hamming weight (number of 1's) of the output sequence assigned to the branch, β: the Hamming weight of the input sequence causing the state change designated by the branch) as in Fig. 9.11(c2), we can regard the state diagram as a signal flow graph and apply the Mason's gain formula[P-3] to get the overall gain from the input all-zero state a to the output all-zero state a' as

$$
\begin{aligned}
T(D,I,J) &= \frac{1}{\Delta}\sum_i M_i\Delta_i \\
&= \frac{\sum \text{All products of the forward gain } M_i \text{ and } \Delta_i \text{ for the diagram without forward path } i}{1-\sum \text{All loop gains} + \sum \text{All products of 2 nontouching loop gains} - \cdots} \\
&= \frac{D^5IJ^3(1-DIJ)+D^6I^2J^4}{1-DIJ-DIJ^2-D^2I^2J^3+D^2I^2J^3} = \frac{D^5IJ^3}{1-(DIJ+DIJ^2)}
\end{aligned}
\tag{9.4.31}
$$

This can be expanded into a power series as

$$
\begin{aligned}
T(D,I,J) &= D^5IJ^3\{1+(DIJ+DIJ^2)+(DIJ+DIJ^2)^2+\cdots\} \\
&= D^5IJ^3 + D^6I^2J^4 + D^6I^2J^5 + D^7I^3J^5 + \cdots
\end{aligned}
\tag{9.4.32}
$$

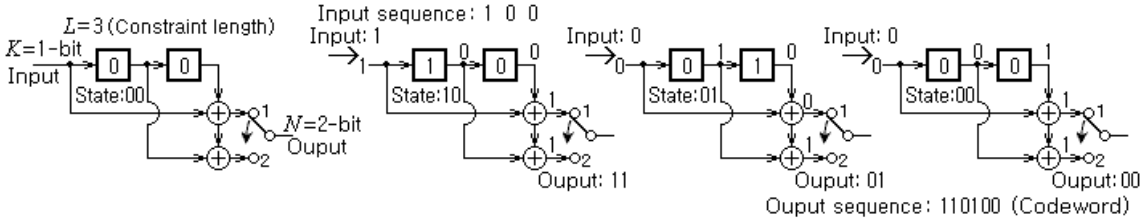
Each term $D^{\alpha}I^{\beta}J^{\gamma}$ of this equation denotes the gain of each forward path starting from the all-zero state and going back to the all-zero state where α is the Hamming weight of the codeword represented by the path, β is the Hamming weight of the input sequence causing the ouput sequence to be generated, and γ is the number of branches contained in the path. In this context, the first term D^5IJ^3 of Eq. (9.4.32) implies that the convolutional encoder in Fig. 9.11(a) may generate a codeword of Hamming weight $\alpha(5)$ and $\gamma \cdot N = 3 \cdot 2 = 6$ bits for the input sequence of Hamming weight $\beta(1)$:

Input:	[1]	[0]	[0]	---- β=1
State:	$a(00) \rightarrow$	$c(10) \rightarrow$	$b(01) \rightarrow$	$e(00)$ ---- γ=3
Output:	11	01	11	---- α=5

$\Rightarrow D^5IJ^3$

The two codewords corresponding to the terms $D^6I^2J^4$ and $D^7I^3J^5$ are

Input:	[1]	[1]	[0]	[0]		[1]	[1]	[1]	[0]	[0]
State:	$a(00)\rightarrow$	$c(10)\rightarrow$	$d(11)\rightarrow$	$b(01)\rightarrow$	$e(00)$	$a(00)\rightarrow$	$c(10)\rightarrow$	$d(11)\rightarrow$	$d(11)\rightarrow$	$b(01)\rightarrow$	$e(00)$
Output:	11	10	10	11		11	10	01	10	11	



(a) Convolutional encoder represented by a schematic diagram and its operation

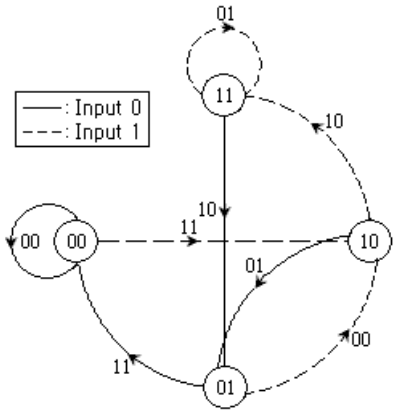
$$G_{N\times LK}=\begin{bmatrix}\mathbf{g}_1\\ \mathbf{g}_2\end{bmatrix}=\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$$\mathbf{o}_1(x)=\mathbf{m}(x)\mathbf{g}_1(x)=(1+0\cdot x+1\cdot x^2)(1+0\cdot x+1\cdot x^2)=1+0\cdot x+0\cdot x^2+0\cdot x^3+1\cdot x^4$$

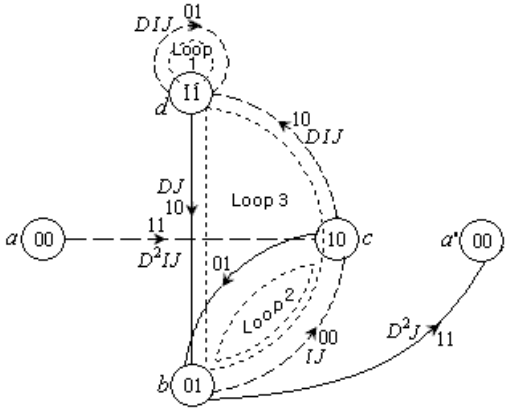
$$\mathbf{o}_2(x)=\mathbf{m}(x)\mathbf{g}_2(x)=(1+0\cdot x+1\cdot x^2)(1+1\cdot x+1\cdot x^2)=1+1\cdot x+0\cdot x^2+0\cdot x^3+1\cdot x^4$$

Output sequence to the input sequence [1 0 1] = [1 1 0 1 0 0 1] (Codeword)

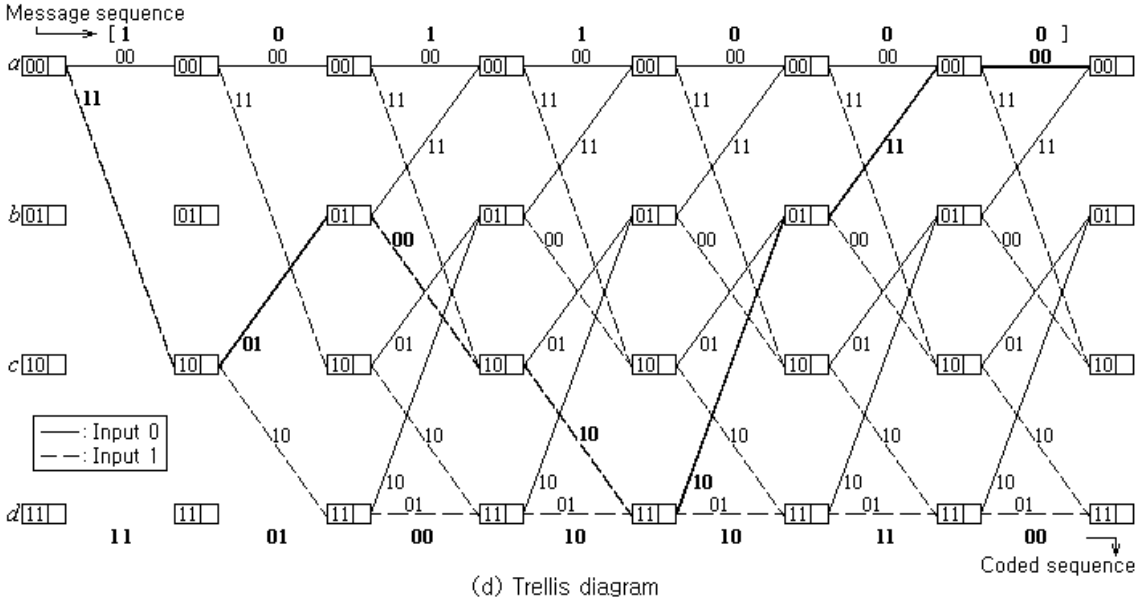
(b) Convolutional encoder represented by a generator polynomial matrix and its operation



(c1) State diagram



(c2) Augmented state diagram with the initial and final all-zero state distinguished and $D^{\alpha}I^{\beta}J$ assigned to each branch (β/α: Hamming weights of input/output)



(d) Trellis diagram

Figure 9.11 Various equivalent representations of a convolutional encoder

The minimum degree in D of the transfer function, that is $d_{\min}=5$ in the case of Eq. (9.4.32), is the *free distance* of the convolutional code, which is the *minimum* (*Hamming*) *distance* among the codewords in the code. Note that the free distance as well as the decoding algorithm heavily affects the BER performance of a convolutional code.

Fig. 9.11(d) shows another representation of the convolutional code called a *trellis*, which can be viewed as a plot of state diagram being developed along the time. This diagram consists of the columns of $2^{(L-1)K}$ state nodes per clock cycle and the solid/dashed branches representing the state transition caused by input 0/1 as depicted in the state diagram (Fig. 9.11(c)) where the $N(2)$-bit output of the encoder to the input is attatched to each branch. In this trellis diagram, each path starting from the initial all-zero state and ending at another all-zero state represents a codeword. From the trellis shown in Fig. 9.11, we can see the shortest two codewords as

Input:	[1]	[0]	[0]		Input:	[1]	[1]	[0]	[0]
State:	$a(00)$	$\rightarrow c(10)$	$\rightarrow b(01)$	$\rightarrow a(00)$	State:	$a(00)$	$\rightarrow c(10)$	$\rightarrow d(11)$	$\rightarrow b(01)$ $\rightarrow a(00)$
Output:	11	01	11		Output:	11	10	10	11

These correspond to the first two terms D^5IJ^3 and $D^6I^2J^4$ of Eq. (9.4.32), respectively.

<**Viterbi Decoding of a Convolutional Coded Sequence**>

As a way of decoding a convolutional coded sequence, an ML (maximum-likelihood) decoding called the *Viterbi algorithm* (VA)[F-1] is most widely used where an ML decoding finds the code sequence that is most likely to have been the input to the encoder at XMTR yielding the received code sequence. Given a received (DTR output or decoder input) sequence $\mathbf{r}=\mathbf{m}_c+\mathbf{e}$ ($\mathbf{m}_c$: a convolutional coded sequence, $\mathbf{e}$: an error), the *Viterbi algorithm* with *soft-decision* searches the trellis diagram for an all-zero-state-to-all-zero-state path whose (encoder) output sequence is closest to $\mathbf{r}$ in terms of Euclidean distance and takes the encoder input sequence corresponding to the 'optimal' path to be the most likely message sequence, while the Viterbi algorithm with *hard-decision* first slices $\mathbf{r}$ to make $\mathbf{r}_q$ consisting of 0 or 1 and then finds an all-zero-state-to-all-zero-state path whose (encoder) output sequence is closest to $\mathbf{r}_q$ in terms of Hamming distance.

The procedure of the Viterbi algorithm is as follows (see Fig. 9.12):

0. To each set of nodes (states), assign the depth level index starting from $l=0$ for the leftmost stage. Divide the decoder input sequence $\mathbf{r}_q$ (hard, i.e. sliced to 0/1) or $\mathbf{r}$ (soft) into N-bit subsequences and distribute each N-bit subsequence to the stages between the node sets.
1. To each branch, attach the (Hamming or Euclidean) distance between the N-bit encoder ouput and the N-bit subsequence of $\mathbf{r}$ or $\mathbf{r}_q$ (distributed to the stage that the branch belongs to) as the *branch cost* (*metric*).
2. With the level index initialized to $l=0$, assign 0 to the initial all-zero state node as its *node cost* (*metric*).
3. Move right by one stage by increasing the level index by one. To every node (at the level) that is connected via some branch(es) to node(s) at the previous level, assign the node cost that is computed by adding the branch cost(s) to the left-hand node cost(s) and taking the minimum if there are multiple branches connected to the node. In the case of multiple branches connected to a node, remove other branches than the survivor branch yielding the minimum path cost.
4. If all the N-bit subsequence of $\mathbf{r}$ or $\mathbf{r}_q$ are not processed, go back to step 3 to repeat the procedure; otherwise, go to the next step 5.
5. Starting from the final all-zero state or the best node with minimum cost, find the optimal path consisting of only the survivor paths and take the encoder input sequence supposedly having caused the output sequence (associated with the optimal path) to be the ML decoded message sequence.

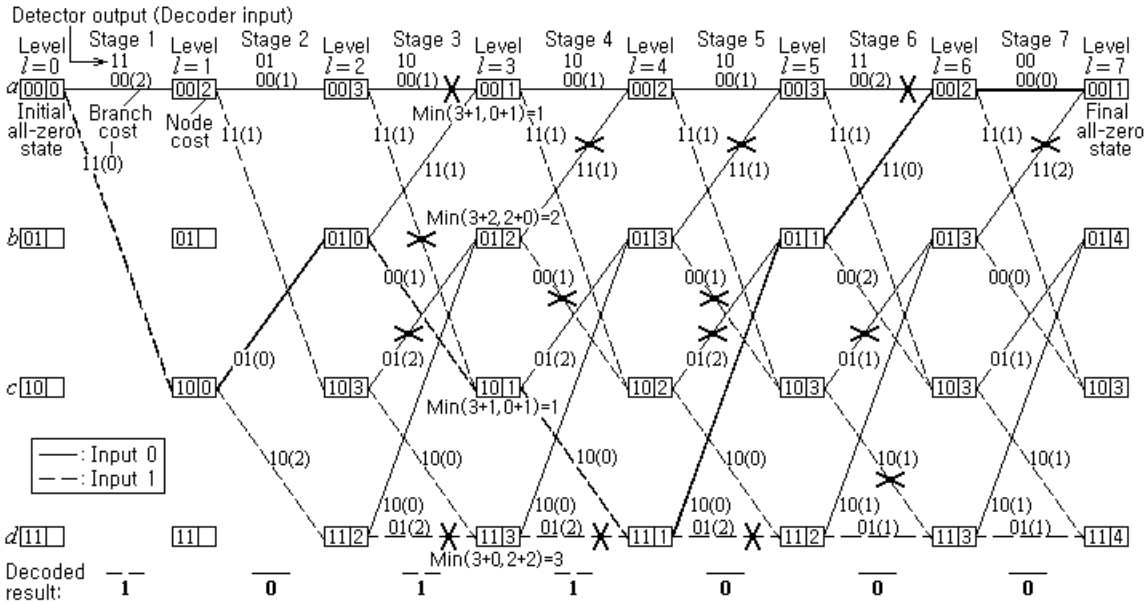


Figure 9.12 The trellis diagram and Viterbi decoding

Fig. 9.12 illustrates a typical decoding procedure using the trellis based on the Viterbi algorithm (VA) stated above where the detector output (code) sequence is [11011010101100]. Note that the removed branches other than the survivor branches are crossed and the optimal path denoted by a series of thick solid/dotted lines (with encoder input 0/1) corresponds to the encoder input sequence [1011000], which has been taken as the decoded result.

Given the generator polynomial matrix G together with the number K of input bits and the channel-DTR output sequence 'detected' as its input arguments, the MATLAB routine 'vit_decoder(G,K,detected)' constructs the trellis diagram and applies the Viterbi algorithm to find the maximum-likelihood decoded message sequence. The following MATLAB program "do_vitdecoder.m" uses the routine 'conv_encoder()' to make a convolutional coded sequence for a message and uses "vit_decoder()" to decode it to recover the original message.

```
%do_vitdecoder.m
% Try using conv_encoder()/vit_decoder()
clear, clf
msg=[1 0 1 1 0 0 0];  % msg=randint(1,100)
lm=length(msg); % Message and its length
G=[1 0 1;1 1 1]; % N x LK Generator polynomial matrix
K=1; N=size(G,1); % Size of encoder input/output
potbe=0.02; % Probability of transmitted bit error
% Use of conv_encoder()/vit_decoder()
ch_input=conv_encoder(G,K,msg) % Self-made convolutional encoder
notbe=ceil(potbe*length(ch_input));
error_bits=randerr(1,length(ch_input),notbe);
detected= rem(ch_input+error_bits,2); % Received/modulated/detected
decoded= vit_decoder(G,K,detected)
noe_vit_decoder=sum(msg~=decoded(1:lm))
```

```
function decoded_seq=vit_decoder(G,K,detected,opmode,hard_or_soft)
% performs the Viterbi algorithm on detected to get the decoded_seq
% G: N x LK Generator polynomial matrix
% K: Number of encoder input bits
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
detected = detected(:).';
if nargin<5|hard_or_soft(1)=='h', detected=(detected>0.5);  end
[N,LK]=size(G);
if rem(LK,K)~=0, error('Column size of G must be a multiple of K'); end
tmp= rem(length(detected),N);
if tmp>0,  detected=[detected  zeros(1,N-tmp)];  end
b=LK-K; % Number of bits representing the state
no_of_states=2^b;   N_msgsymbol=length(detected)/N;
for m=1:no_of_states
   for n=1:N_msgsymbol+1
      states(m,n)=0; % inactive in the trellis
      p_state(m,n)=0; n_state(m,n)=0; input(m,n)=0;
   end
end
states(1,1)=1; % make the initial state active
cost(1,1)=0; K2=2^K;
for n=1:N_msgsymbol
 y=detected((n-1)*N+1:n*N); % Received sequence
 n1=n+1;
 for m=1:no_of_states
  if states(m,n)==1 % active
    xb=deci2bin1(m-1,b);
    for m0=1:K2
      u=deci2bin1(m0-1,K);
      [nxb(m0,:),yb(m0,:)]=state_eq(xb,u,G);
      nxm0=bin2deci(nxb(m0,:))+1;
      states(nxm0,n1)=1;
      dif=sum(abs(y-yb(m0,:)));
      d(m0)=cost(m,n)+dif;
      if p_state(nxm0,n1)==0 % Unchecked state node?
        cost(nxm0,n1)=d(m0);
        p_state(nxm0,n1)=m; input(nxm0,n1)=m0-1;
       else
        [cost(nxm0,n1),i]=min([d(m0) cost(nxm0,n1)]);
        if i==1, p_state(nxm0,n1)=m; input(nxm0,n1)=m0-1; end
      end
    end
  end
 end
end
decoded_seq=[];
if nargin>3 & ~strncmp(opmode,'term',4)
  [min_dist,m]=min(cost(:,n1)); % Trace back from best-metric state
 else m=1; % Trace back from the all-0 state
end
for n=n1:-1:2
  decoded_seq= [deci2bin1(input(m,n),K) decoded_seq];
  m=p_state(m,n);
end
```

The following program "do_vitdecoder1.m" uses the Communication Toolbox functions 'convenc()' and 'vitdec()' where 'vitdec()' is used several times with different input argument values to show the readers its various usages. Now, it is time to see the usage of the function 'vitdec()'.

```
%do_vitdecoder1.m
% shows various uses of Communication Toolbox function convenc()
% with KxN Code generator matrix Gc - octal polynomial representation
clear, clf
%msg=[1 0 1 1 0 0 0];
msg=randint(1,100)
lm=length(msg); % Message and its length
potbe=0.02; % Probability of transmitted bit error
Gc=[5 7]; % 1 0 1 -> 5, 1 1 1 -> 7 (octal number)
Lc=3; % 1xK constraint length vector for each input stream
[K,N]=size(Gc); % Number of encoder input/output bits
trel=poly2trellis(Lc,Gc); % Trellis structure
ch_input1=convenc(msg,trel); % Convolutional encoder
notbe1=ceil(potbe*length(ch_input1));
error_bits1=randerr(1,length(ch_input1),notbe1);
detected1= rem(ch_input1+error_bits1,2); % Received/modulated/detected
% with hard decision
Tbdepth=3; % Traceback depth
decoded1= vitdec(detected1,trel,Tbdepth,'trunc','hard')
noe_vitdec_trunc_hard=sum(msg~=decoded1(1:lm))
decoded2= vitdec(detected1,trel,Tbdepth,'cont','hard');
noe_vitdec_cont_hard=sum(msg(1:end-Tbdepth)~=decoded2(Tbdepth+1:end))
% with soft decision
ncode= [detected1+0.1*randn(1,length(detected1)) zeros(1,Tbdepth*N)];
quant_levels=[0.001,.1,.3,.5,.7,.9,.999];
NSDB=ceil(log2(length(quant_levels))); % Number of Soft Decision Bits
qcode= quantiz(ncode,quant_levels); % Quantized
decoded3= vitdec(qcode,trel,Tbdepth,'trunc','soft',NSDB);
noe_vitdec_trunc_soft=sum(msg~=decoded3(1:lm))
decoded4= vitdec(qcode,trel,Tbdepth,'cont','soft',NSDB);
noe_vitdec_cont_soft=sum(msg~=decoded4(Tbdepth+1:end))
% Repetitive use of vitdec() to process the data block by block
delay=Tbdepth*K; % Decoding delay depending on the traceback depth
% Initialize the message sequence, decoded sequence,
%   state metric, traceback state/input, and encoder state.
msg_seq=[]; decoded_seq=[];
m=[]; s=[]; in=[]; encoder_state=[];
N_Iter=100;
for itr=1:N_Iter
   msg=randint(1,1000); % Generate the message sequence in a random way
   msg_seq= [msg_seq msg]; % Accumulate the message sequence
   if itr==N_Iter, msg=[msg zeros(1,delay)]; end % Append with zeros
   [coded,encoder_state]=convenc(msg,trel,encoder_state);
   [decoded,m,s,in]=vitdec(coded,trel,Tbdepth,'cont','hard',m,s,in);
   decoded_seq=[decoded_seq decoded];
end
lm=length(msg_seq);
noe_repeated_use=sum(msg_seq(1:lm)~=decoded_seq(delay+[1:lm]))
```

<Usage of the Viterbi Decoding Function 'vitdec()' with 'convenc()' and 'poly2trellis()'>

To apply the MATLAB functions 'convenc()'/'vitdec()', we should first use 'poly2trellis()' to build the trellis structure with an 'octal code generator' describing the connections among the inputs, registers, and outputs. Fig. 9.13 illustrates how the octal code generator matrix G_c as well as the binary generator matrix G and the constraint length vector L_c is constructed for a given convolutional encoder. An example of using 'poly2trellis()' to build the trellis structure for 'convenc()'/'vitdec()' is as follows:

```
trellis=poly2trellis(Lc,Gc);
```

Here is a brief introduction of the usages of the Communication Toolbox functions 'convenc()' and 'vitdec()'. See the MATLAB Help manual or The Mathworks webpage[W-7] for more details.

(1) `coded=convenc(msg,trellis);`

`msg`: A message sequence to be encoded with a convolutional encoder described by '`trellis`'.

(2) `decoded=vitdec(coded,trellis,tbdepth,opmode,dectype,NSDB);`

`coded` : A convolutional coded sequence possibly corrupted by a noise. It should consist of binary numbers (0/1), real numbers between 1(logical zero) and -1(logical one), or integers between 0 and 2^{NSDB}-1 (NSDB: the number of soft-decision bits given as the optional 6th input argument) corresponding to the quantization level depending on which one of {'hard', 'unquant', 'soft'} is given as the value of the fifth input argument 'dectype' (decision type).

`trellis` : A trellis structure built using the MATLAB function 'poly2trellis()'.

`tbdepth`: Traceback depth (length), i.e., the number of trellis branches used to construct each traceback path. It should be given as a positive integer, say, about five times the constraint length. In case the fourth input argument 'opmode' (operation mode) is 'cont' (continuous), it causes the decoding delay, i.e., the number of zero symbols preceding the first decoded symbol in the output 'decoded' and as a consequence, the decoded result should be advanced by tbdepth * K where K is the number of encoder input bits.

`opmode`: Operation mode of the decoding prosess. If it is set to 'cont' (continuous mode), the internal state of the decoder will be saved for use with the next frame. If it is set to 'trunc' (truncation mode), each frame will be processed independently, and the traceback path starts at the best-metric state and always ends in the all-zero state. If it is set to 'term' (termination mode), each frame is treated independently, and the traceback path always starts and ends in the all-zero state. This mode is appropriate when the uncoded message signal has enough zeros, say, K * Max(Lc)-1) zeros at the end of each frame to fill all memory registers of the encoder.

`dectype`: Decision type. It should be set to 'unquant', 'hard', or 'soft' depending on the characteristic of the input coded sequence (coded) as follows:

- 'hard' (decision) when the coded sequence consists of binary numbers 0 or 1.
- 'unquant' when the coded sequence consists of real numbers between -1(logical 1) and +1(logical 0).
- 'soft' (decision) when the optional 6th input argument NSDB is given and the coded sequence consists of integers between 0 and 2^{NDSB}-1 corresponding to the quantization level.

NSDB : Number of software decision bits used to represent the input coded seqeuence. It is needed and active only when dectype is set to 'soft'.

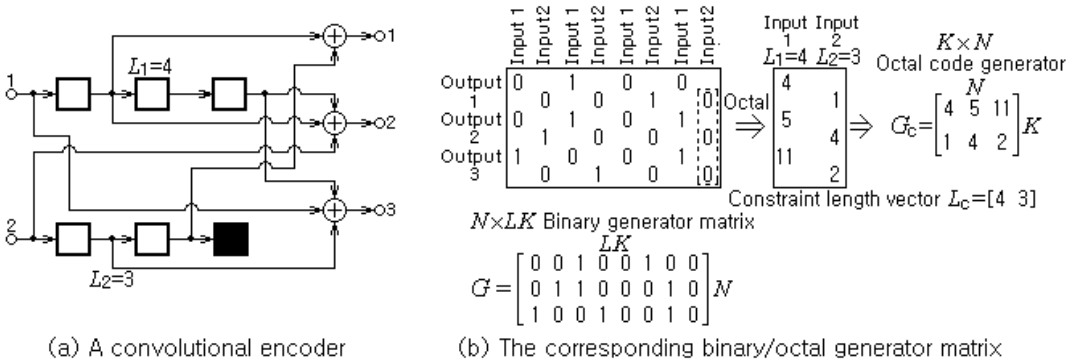


Figure 9.13 A convolutional encoder and the corresponding binary/octal generator matrix

(3) `[decoded,m,s,in]=vitdec(code,trellis,tbdepth,opmode,dectype,m,s,in)`

This format is used for a repetitive use of 'vitdec()' with the continuous operation mode where the state metric 'm', traceback state 's', and traceback input 'in' are supposed to be initialized to empty sets at first and then handed over successively to the next iteration.

See the above program "do_vitdecoder1.m" or the MATLAB help manual for some examples of using 'convenc()' and 'vitdec()'.

9.4.5 Trellis-Coded Modulation (TCM)

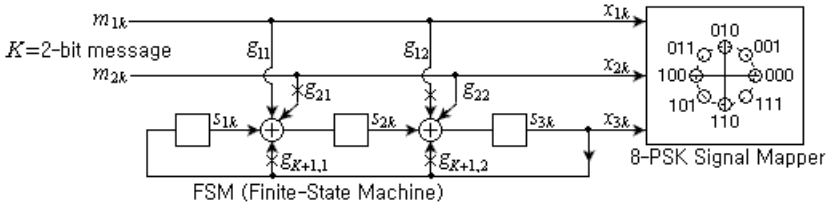
By channel coding discussed in the previous sections, additional redundant bits for error control are transmitted in the code and as a consequence, wider bandwidth is needed to keep the same data rate while lower BER can be obtained with the same SNR or the SNR to obtain the same BER can be decreased. This shows a trade-off between bandwidth efficiency and power efficiency. As an approach to mediate between these two conflicting factors, TCM (trellis-coded modulation) was invented by Gottfried Ungerboeck[U-1, U-2] . It is just like a marriage between modulation residing on the constellation and coding living on the trellis toward a more effective utilization of bandwidth and power. A (code) rate-2/3 TCM encoder combining an $(N,K)=(3,2)$ convolutional coding and an 2^3=8-PSK modulation is depicted in Fig. 9.14(a). In this encoder, the 8-PSK signal mapper generates one of the eight PSK signals depending on the 3-bit symbol $\mathbf{x}_k=[x_{1k}\ x_{2k}\ x_{3k}]$ that consists of two (uncoded) message bits x_{1k} and x_{2k} and an additional coded bit that is the output of the FSM (finite-state machine). Since the encoding scheme can be described by a trellis, the Viterbi algorithm can be used to decode the TCM coded signal. There are a couple of things to mention about TCM:

- The number of signal points in the constellation is larger compared with the uncoded case. For example, the constellation size or modulation order used by the TCM scheme of Fig. 9.14(a) is $2^{K+1}=8$. This is two times as large as that of QPSK modulation that would be used to send the message by $K=2$ -bit symbols with no coding. Note that a larger modulation order with the same SNR decreases the minimum distance among the codewords so that the BER may suffer.
- TCM has some measures to combat the possible BER performance degradation problem:
  - ◦ The convolutional coding allows only certain sequences (paths) of signal points so that the free distance among different signal paths in the trellis can be increased.

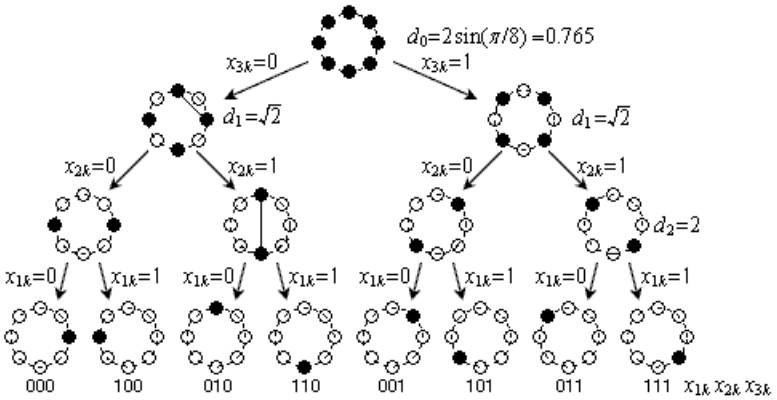
◦ The TCM decoder uses soft-decision decoding to find the path with minimum (squared) Euclidean distance through the trellis. This makes the trellis code design trying to maximize the Euclidean distance among the codewords.

◦ As illustrated in Fig. 9.14(b1) and (b2), the *set partioning* let the more significant message bit(s) have a larger Euclidean distance from its complement so that there are less likely errors in the uncoded message bits than in the coded bit(s).

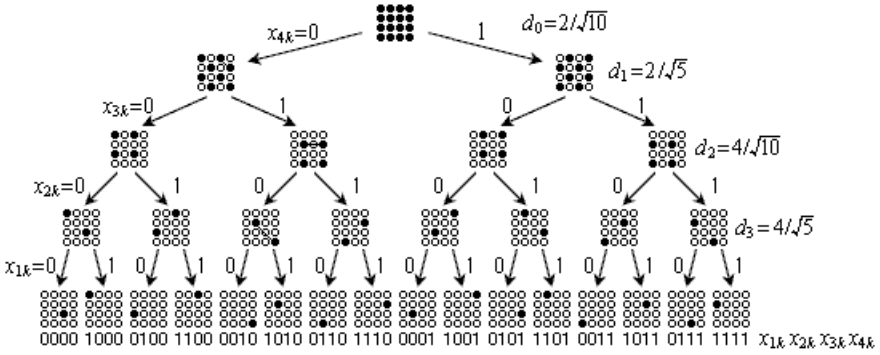
◦ The FSM of TCM encoder has some inputs added to the adders between the shift registers. Therefore it seems that TCM encoding/decoding can not be implemented using the general convolutional encoder/decoder, requiring its own encoder/decoder rountines of every TCM encoder/decoder.



(a) TCM encoder consisting of a 2^3-state FSM and an 8-PSK signal mapper



(b1) Set partitioning of an 8-PSK signal set into subsets with increasing minimum distance



(b2) Set partitioning of a 16-QAM signal set into subsets with increasing minimum distance

Figure 9.14 An example of TCM (Trellis-Coded Modulation) coding

The following program "sim_TCM.m" uses the two subroutines "TCM_encoder()" and "TCM_decoder()" to simulate the TCM encoding (shown in Fig. 9.14(a)) and the corresponding TCM decoding.

```
%sim_TCM.m
% simulates a Trellis-coded Modulation
clear, clf
lm=1e4; msg=randint(1,lm);
K=2; N=K+1; Ns=3; % Size of encoder input/output/state
M=2^N; Constellation=exp(j*2*pi/M*[0:M-1]); % Constellation
ch_input=TCM_encoder('TCM_state_eq0',K,Ns,N,msg,Constellation);
lc= length(ch_input);
SNRbdB=5; SNRdB=SNRbdB+10*log10(K); % SNR per K-bit symbol
sigma=1/sqrt(10^(SNRdB/10));
noise=sigma/sqrt(2)*(randn(1,lc)+j*randn(1,lc)); % Complex noise
received = ch_input + noise; var(received)
received = awgn(ch_input,SNRdB); var(received) % Alternative
decoded_seq=TCM_decoder('TCM_state_eq0',K,Ns,received,Constellation);
ber_TCM8PSK = sum(decoded_seq(1:lm)~=msg)/lm
ber_QPSK_theory = prob_error(SNRbdB,'PSK',K,'BER')
```

```
function [output,state]
        =TCM_encoder(state_eq,K,Ns,N,input,state,Constellation,opmode)
% Generates the output sequence of a binary TCM encoder
% Input: state_eq = External function for state eqn saved in an M-file
%        K    = Number of input bits entering the encoder at each cycle
%        Nsb  = Number of state bits of the TCM encoder
%        N    = Number of output bits of the TCM encoder
%        input= Binary input message seq.
%        state= State of the conv_encoder
%        Constellation= Signal sets for signal mapper
% Output: output= Sequence of signal points on Contellation
%         state = Updated state
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
tmp= rem(length(input),K);
input= [input zeros(1,(K-tmp)*(tmp>0))];
if nargin<6, state=zeros(1,Nsb);
 elseif length(state)==2^N, Constellation=state; state=zeros(1,Nsb);
end
input_length= length(input);  N_msgsymbol= input_length/K;
input1= reshape(input,K,N_msgsymbol).';
outputs= [];
for l=1:N_msgsymbol
   ub= input1(l,:);
   [state,output]=feval(state_eq,state,ub,Constellation);
   outputs= [outputs output];
end
```

```
function [s1,x]=TCM_state_eq0(s,u,Constellation)
% State equation for the TCM_encoder in Fig. 9.14(a)
% Input:  s= State, u= Input, Constellation
% Output: s1= Next state, x= Output
s1= [s(3) rem([s(1)+u(1) s(2)+u(2)],2)];
x= Constellation(bin2deci([u(1) u(2) s(3)])+1);
```

```
function decoded_seq
            =TCM_decoder(state_eq,K,Nsb,received,Constellation,opmode)
% Performs the Viterbi algorithm on the PSK demodulated signal
% Input: state_eq = External function for state eqn saved in an M-file
%        K   = Number of input bits entering the encoder at each cycle.
%        Nsb = Number of state bits of the TCM encoder
%        received = received sequence
%        Constellation: Signal sets for signal mapper
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
N_states=2^Nsb;
N_msgsymbol=length(received);
for m=1:N_states
   for n=1:N_msgsymbol+1
      states(m,n)=0; %inactive in the trellis diagram
      p_state(m,n)=0; n_state(m,n)=0; input(m,n)=0;
   end
end
states(1,1)=1; % make the initial state active
cost(1,1)=0; K2=2^K;
for n=1:N_msgsymbol
  y=received(n); %received sequence
  n1=n+1;
  for m=1:N_states
    if states(m,n)==1 %active
      xb=deci2bin1(m-1,Nsb);
      for m0=1:K2
        u=deci2bin1(m0-1,K);
        [nxb(m0,:),yb(m0)]=feval(state_eq,xb,u,Constellation);
        nxm0=bin2deci(nxb(m0,:))+1;  states(nxm0,n1)=1;
        % Accumulated squared Euclidean distance as path-to-node cost
        difference=y-yb(m0);
        d(m0)=cost(m,n)+difference*conj(difference);
        if p_state(nxm0,n1)==0
          cost(nxm0,n1)=d(m0);
          p_state(nxm0,n1)=m; input(nxm0,n1)=m0-1;
         else
          [cost(nxm0,n1),i]=min([d(m0) cost(nxm0,n1)]);
          if i==1, p_state(nxm0,n1)=m; input(nxm0,n1)=m0-1; end
        end
      end
    end
  end
end
decoded_seq=[];
if nargin<6 | ~strncmp(opmode,'term',4)
    % trace back from the best-metric state (default)
   [min_cost,m]=min(cost(:,n1));
  else m=1; % trace back from the all-0 state
end
for n=n1:-1:2
  decoded_seq= [deci2bin1(input(m,n),K) decoded_seq];
  m=p_state(m,n);
end
```

9.4.6 Turbo Coding

In order for a linear block code or a convolutional code to approach the theoretical limit imposed by Shannon's channel capacity (see Eq. (9.3.16) or Fig. 9.7) in terms of bandwidth/power efficiency, its codeword or constraint length should be increased to such an intolerable degree that the maximum likelihood decoding can become unrealizable. Possible solutions to this dilemma are two classes of powerful error correcting codes, each called turbo codes and LDPC (lower-density parity-check) codes, that can achieve a near-capacity (or near-Shannon-limit) performance with a reasonable complexity of decoder. The former is the topic of this section and the latter will be introduced in the next section.

Fig. 9.15(a) shows a turbo encoder consisting of two *recursive systematic convolutional* (RSC) encoders and an interleaver where the interleaver permutes the message bits in a random way before input to the second encoder. (Note that the modifier '*systematic*' means that the uncoded message bits are imbedded in the encoder output stream as they are.) The code rate will be 1/2 or 1/3 depending on whether the puncturing is performed or not. (Note that puncturing is to omit transmitting some coded bits for the purpose of increasing the code rate beyond that resulting from the basic structure of the encoder.) Fig. 9.15(b) shows a demultiplexer, which classifies the coded bits into two groups, one from encoder 1 and the other from encoder 2, and applies each of them to the corresponding decoder. Fig. 9.15(c) shows a turbo decoder consisting of two decoders concatenated and separated by an interleaver where one decoder processes the systematic (message) bit sequence y^s and the parity bit sequence y^{1p}/y^{2p} together with the extrinsic information L_{ej} (provided by the other decoder) to produce the information L_{ei} and provides it to the other decoder in an iterative manner. The turbo encoder and the demultiplexer are cast into the MATLAB routines 'encoderm()' and 'demultiplex()', respectively. Now, let us see how the two types of decoder, each implementing the log-MAP (maximum a posteriori probability) algorithm and the SOVA (soft-out Viterbi algorithm), are cast into the MATLAB routines 'logmap()' and 'sova()', respectively.

<Log-MAP (Maximum a Posteriori Probability) Decoding cast into 'logmap()'>

To understand the operation of the turbo decoder, let us begin with the definition of *priori LLR* (*log-likelihood ratio*), called a priori L-value, which is a soft value measuring how high the probability of a binary random variable $\mathbf{u}$ being +1 is in comparison with that of $\mathbf{u}$ being -1:

$$L_{\mathbf{u}}(u) = \ln \frac{P_{\mathbf{u}}(u=+1)}{P_{\mathbf{u}}(u=-1)} \quad \text{with } P_{\mathbf{u}}(u) \text{: the probability of } \mathbf{u} \text{ being } u \tag{9.4.33}$$

This is a priori information known before the result $\mathbf{y}$ caused by $\mathbf{u}$ becomes available**.** While the sign of LLR

$$\hat{u} = \text{sign}\{L_{\mathbf{u}}(u)\} = \begin{cases} +1 & P_{\mathbf{u}}(u=+1) > P_{\mathbf{u}}(u=-1) \\ -1 & P_{\mathbf{u}}(u=+1) < P_{\mathbf{u}}(u=-1) \end{cases} \tag{9.4.34}$$

is a hard value denoting whether or not the probability of $\mathbf{u}$ being +1 is higher than that of $\mathbf{u}$ being -1, the magnitude of LLR is a soft value describing the reliability of the decision based on $\hat{u}$. Conversely, $P_{\mathbf{u}}(u=+1)$ and $P_{\mathbf{u}}(u=-1)$ can be derived from $L_{\mathbf{u}}(u)$:

$$P_{\mathbf{u}}(u=+1) \overset{(9.4.33)}{=} e^{L(u)} P_{\mathbf{u}}(u=-1) \overset{P(u=+1)+P(u=-1)=1}{\longrightarrow} P_{\mathbf{u}}(u=+1) = \frac{e^{L(u)}}{1+e^{L(u)}} \text{ and } P_{\mathbf{u}}(u=-1) = \frac{1}{1+e^{L(u)}}$$

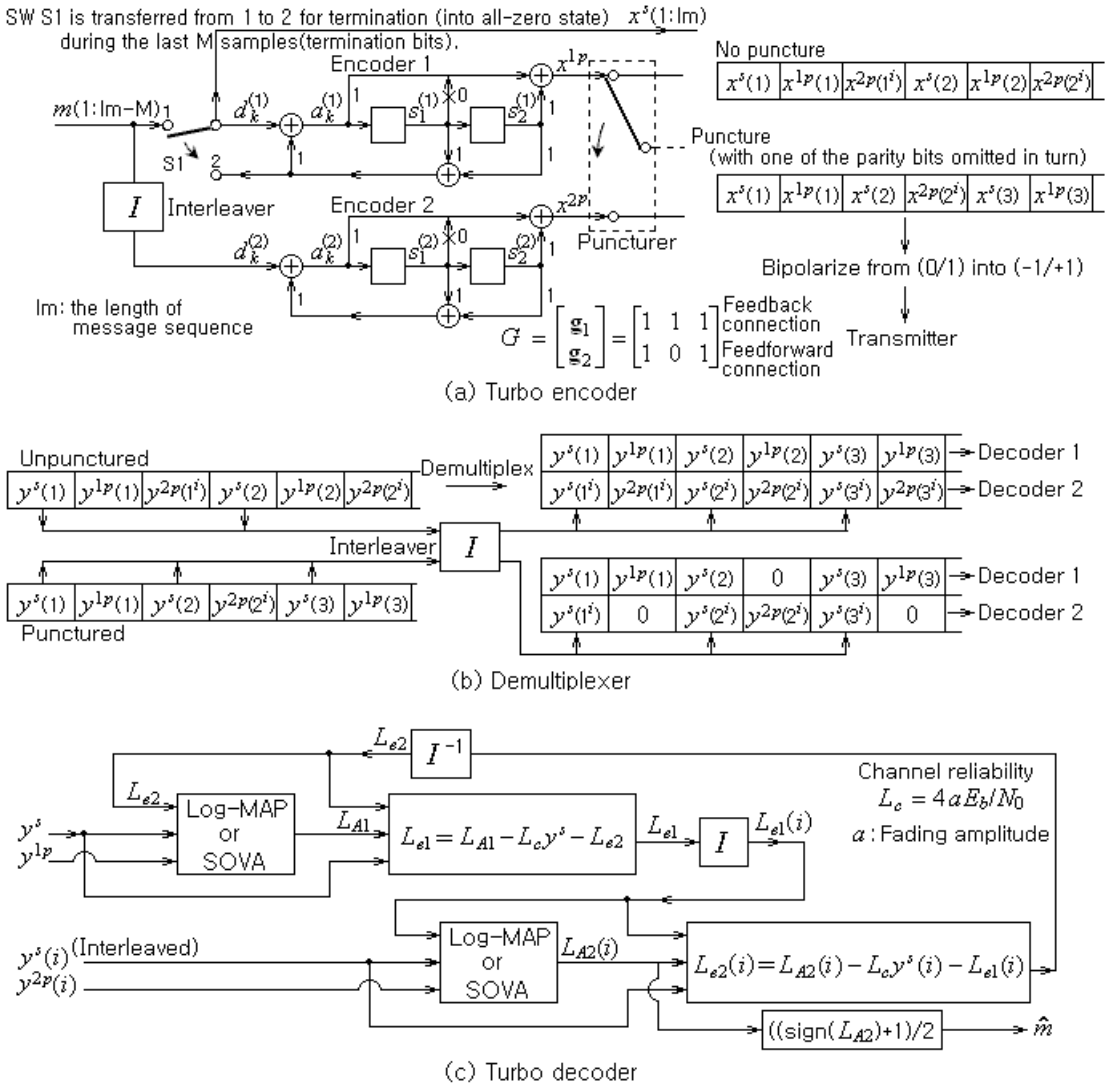


Figure 9.15 A typical turbo coding

```
function y = demultiplexer(r,map,puncture)
%Copyright 1998, Yufei Wu, MPRG lab, Virginia Tech. for academic use
% map: Interleaver mapping
Nb = 3-puncture; lu = length(r)/Nb;
if puncture==0   % unpunctured
  for i=1:lu, y(:,2*i) = r(3*i-[1 0]).'; end
else             % punctured
  for i=1:lu
     i2 = i*2;
     if rem(i,2)>0, y(:,i2)=[r(i2); 0]; else y(:,i2)=[0; r(i2)];  end
  end
end
sys_bit_seq = r(1,1:Nb:end); % the systematic bits for both decoders
y(:,1:2:lu*2) = [sys_bit_seq; sys_bit_seq(map)];
```

```
function x = rsc_encode(G,m,termination)
% Copyright 1998, Yufei Wu, MPRG lab, Virginia Tech. for academic use
% encodes a binary data block m (0/1) with a RSC (recursive systematic
% convolutional) code defined by generator matrix G, returns the output
% in x (0/1), terminates the trellis with all-0 state if termination>0
if nargin<3, termination = 0; end
[N,L] = size(G); % Number of output bits, Constraint length
M = L-1; % Dimension of the state
lu = length(m)+(termination>0)*M; % Length of the input
lm = lu-M; % Length of the message
state = zeros(1,M); % initialize the state vector
% To generate the codeword
x = [];
for i = 1:lu
   if termination<=0 | (termination>0 & i<=L_info)
     d_k = m(i);
    elseif termination>0 & i>lm
     d_k = rem(G(1,2:L)*state.',2);
   end
   a_k = rem(G(1,:)*[d_k state].',2);
   xp = rem(G(2,:)*[a_k state].',2); % 2nd output (parity) bits
   state = [a_k state(1:M-1)]; % Next sttate
   x = [x [d_k; xp]]; % since systematic, first output is input bit
end
```

```
function x = encoderm(m,G,map,puncture)
% Copyright 1998, Yufei Wu, MPRG lab, Virginia Tech. for academic use
% map: Interleaver mapping
% If puncture=0(unpunctured), it operates with a code rate of 1/3.
% If puncture>0(punctured), it operates with a code rate of 1/2.
% Multiplexer chooses odd/even-numbered parity bits from RSC1/RSC2.
[N,L] = size(G); % Number of output pits, Constraint length
M = L-1; % Dimension of the state
lm = length(m); % Length of the information message block
lu = lm + M; % Length of the input sequence
% 1st RSC coder output
x1 = rsc_encode(G,m,1);
% interleave input to second encoder
mi = x1(1,map);  x2 = rsc_encode(G,mi,0);
% parallel to serial multiplex to get the output vector
x = [];
if puncture==0   % unpunctured, rate = 1/3;
  for i=1:lu
     x = [x x1(1,i) x1(2,i) x2(2,i)];
  end
 else            % punctured into rate 1/2
  for i=1:lu
     if rem(i,2), x = [x x1(1,i) x1(2,i)]; % odd parity bits from RSC1
      else  x = [x x1(1,i) x2(2,i)];       % even parity bits from RSC2
     end
  end
end
x = 2*x - 1; % into bipolar format (+1/-1)
```

This can be expressed as

$$P_{\mathbf{u}}(u)=\frac{e^{(u+1)L(u)/2}}{1+e^{L(u)}}=\begin{cases}e^{L(u)}/(1+e^{L(u)}) & \text{for } u=+1\\ 1/(1+e^{L(u)}) & \text{for } u=-1\end{cases} \tag{9.4.35}$$

Also, we define the *conditioned LLR*, which is used to detect the value of $\mathbf{u}$ based on the value of another random variable $\mathbf{y}$ affected by $\mathbf{u}$, as the *LAPP* (*Log A Posteriori Probability*):

$$L_{\mathbf{u}|\mathbf{y}}(u\mid y)=\ln\frac{P_{\mathbf{u}}(u=+1|y)}{P_{\mathbf{u}}(u=-1|y)}\overset{(2.1.4)}{=}\ln\frac{P(y|u=+1)P_{\mathbf{u}}(u=+1)/P(y)}{P(y|u=-1)P_{\mathbf{u}}(u=-1)/P(y)}=\ln\frac{P(y|u=+1)}{P(y|u=-1)}+\ln\frac{P_{\mathbf{u}}(u=+1)}{P_{\mathbf{u}}(u=-1)} \tag{9.4.36}$$

Now, let $\mathbf{y}$ be the output of a fading AWGN (additive white Gaussian noise) channel (with fading amplitude a and SNR per bit E_b/N_0) given $\mathbf{u}$ as the input. Then, this equation for the conditioned LLR can be written as

$$L_{\mathbf{u}|\mathbf{y}}(u\mid y)=\ln\frac{\exp(-(E_b/N_0)(y-a)^2)}{\exp(-(E_b/N_0)(y+a)^2)}+\ln\frac{P_{\mathbf{u}}(u=+1)}{P_{\mathbf{u}}(u=-1)}=4ay\frac{E_b}{N_0}+L_{\mathbf{u}}(u)=L_c y+L_{\mathbf{u}}(u) \tag{9.4.37}$$

$$\text{with } L_c=4a\frac{E_b}{N_0}\text{: the } \textit{channel reliability}$$

The objective of *BCJR* (*Bahl-Cocke-Jelinek-Raviv*) *MAP* (*Maximum A posteriori Probability*) *algorithm* proposed in [B-1] is to detect the value of the k th message bit u_k depending on the sign of the following LAPP function:

$$\begin{aligned}L_A(u_k)&=\ln\frac{P_{\mathbf{u}}(u_k=+1|\mathbf{y})}{P_{\mathbf{u}}(u_k=-1|\mathbf{y})}=\ln\frac{\sum_{(s',s)\in S^+}p(s_k=s',s_{k+1}=s,\mathbf{y})/p(\mathbf{y})}{\sum_{(s',s)\in S^-}p(s_k=s',s_{k+1}=s,\mathbf{y})/p(\mathbf{y})}\\&=\ln\frac{\sum_{(s',s)\in S^+}p(s_k=s',s_{k+1}=s,\mathbf{y})}{\sum_{(s',s)\in S^-}p(s_k=s',s_{k+1}=s,\mathbf{y})}\end{aligned} \tag{9.4.38}$$

with S^+/S^-: the set of all the encoder state transitions from s' to s caused by $u_k=+1/-1$

Note that P and p denote the probability of a discrete-valued random variable and the probability density of a continuous-valued random variable. The numerator/denominator of this LAPP function are the sum of the probabilities that the channel output to $u_k=+1/-1$ will be $\mathbf{y}=\{\mathbf{y}_{j<k}, y_k, \mathbf{y}_{j>k}\}$ with the encoder state transition from s' to s where each joint probability density $p(s',s,\mathbf{y})=p(s_k=s',s_{k+1}=s,\mathbf{y})$ can be written as

$$p(s',s,\mathbf{y})=p(s_k=s',s_{k+1}=s,\mathbf{y})=p(s',\mathbf{y}_{j<k})p(s,y_k\mid s')p(\mathbf{y}_{j>k}|s)=\alpha_{k-1}(s')\gamma_k(s',s)\beta_k(s) \tag{9.4.39}$$

where $\alpha_{k-1}(s')=p(s',\mathbf{y}_{j<k})$ is the probability that the state s_k at the k th depth level (stage) in the trellis is s' with the output sequence $\mathbf{y}_{j<k}$ generated before the k th level, $\gamma_k(s',s)=p(s,y_k|s')$ is the probability that the state transition from $s_k=s'$ to $s_{k+1}=s$ is made with the output $\mathbf{y}_k$ generated, and $\beta_k(s)=p(\mathbf{y}_{j>k}|s)$ is the probability that the state s_{k+1} is s with the output sequence

$\mathbf{y}_{j>k}$ generated after the k th level. The first and third factors $\alpha_{k-1}(s')/\beta_k(s)$ can be computed in a forward/backward recursive way:

$$\alpha_k(s) = p(s, \mathbf{y}_{j<k+1}) = p(s', \mathbf{y}_{j<k}, s, y_k) = \sum_{s'\in S} p(s', \mathbf{y}_{j<k}) p(s, y_k | s')$$
$$= \sum_{s'\in S} \alpha_{k-1}(s')\gamma_k(s', s) \text{ with } \alpha_0(0) = 1,\ \alpha_0(s) = 0 \text{ for } s \neq 0 \tag{9.4.40}$$

$$\beta_{k-1}(s') = p(\mathbf{y}_{j>k-1} | s') = p(s', y_k, s, \mathbf{y}_{j>k} | s') = \sum_{s\in S} p(s, y_k | s') p(\mathbf{y}_{j>k} | s)$$
$$= \sum_{s'\in S} \gamma_k(s', s)\beta_k(s) \tag{9.4.41a}$$

with

$$\beta_K(s) = \begin{cases} \beta_K(0)=1 \text{ and } \beta_K(s)=0\ \forall s\neq 0 & \text{if terminated at all-zero state} \\ \beta_K(s)=1/N_s\ \forall s & \text{otherwise} \end{cases} \tag{9.4.41b}$$

where $N_s = 2^{L-1}$ (L: the constraint length) is the number of states and K is the number of decoder input symbols. The second factor $\gamma_k(s', s)$ can be found as

$$\gamma_k(s', s) = p(s, y_k | s') \overset{(2.1.4)}{=} \frac{p(s', s, y_k)}{p(s')} = \frac{p(s', s) p(y_k | s', s)}{p(s')}$$
$$= p(s_{k+1} | s'_k) p(y_k | s'_k, s_{k+1}) = p(u_k)\, p(y_k | u_k) = p(u_k)\, p(y_k^s, y_k^p | u_k, x_k^p(u_k))$$
$$\overset{\substack{(9.4.35) \\ \text{AWGN channel with fading amplitude } a \\ \text{and SNR per bit } E_b/N_0}}{=} \frac{e^{(u+1)L(u)/2}}{1+e^{L(u)}} \exp\left(-\frac{E_b}{N_0}(y_k^s - a u_k)^2 - \frac{E_b}{N_0}(y_k^p - a x_k^p(u_k))^2 \right)$$
$$= \frac{e^{(u+1)L(u)/2}}{1+e^{L(u)}} A_k \exp\left(2a\frac{E_b}{N_0}(y_k^s u_k + y_k^p x_k^p(u_k)) \right) \text{ where } u_k^2 = 1$$
$$= \frac{e^{(u+1)L(u)/2}}{1+e^{L(u)}} A_k \exp\left(\frac{1}{2} L_c [y_k^s \quad y_k^p] \begin{bmatrix} u_k \\ x_k^p(u_k) \end{bmatrix} \right) \tag{9.4.42}$$

with $A_k = \exp\left(-\frac{E_b}{N_0}\left((y_k^s)^2 + a^2 u_k^2 + (y_k^p)^2 + a^2 (x_k^p(u_k))^2 \right) \right)$ and $L_c = 4a\frac{E_b}{N_0}$ (*channel reliability*)

Note a couple of things about this equation:

- To compute γ_k, we need to know the channel fading amplitude a and the SNR per bit E_b/N_0.
- A_k does not have to be computed since it will be substituted directly or via α_k (Eq. (9.4.40)) or β_k (Eq. (9.4.41)) into Eq. (9.4.39), then substituted into both the numerator and the denominator of Eq. (9.4.38), and finally cancelled.

The following MATLAB routine 'logmap()' corresponding to the block named 'Log-MAP or SOVA' in Fig. 9.15(c) uses these equations to compute the LAPP function (9.4.38). Note that in the routine, the multiplications of the exponential terms are done by adding their exponents and that is why Alpha and Beta (each representing the exponent of α_k and β_k) are initialized to a large negative number as –Infty=–100 (corresponding to a nearly-zero $e^{-100} \approx 0$) under the assumption of initial all-zero state and for the termination of decoder 1 in all-zero state, respectively. (Q: Why is Beta initialized to $-\ln Ns$ (-log(Ns)) for non-termination of decoder 2?)

```
function L_A = logmap(Ly,G,Lu,ind_dec)
% Copyright 1998, Yufei Wu, MPRG lab, Virginia Tech. for academic use
% Log_MAP algorithm using straightforward method to compute branch cost
% Input: Ly      = scaled received bits Ly=0.5*L_c*y=(2*a*rate*Eb/N0)*y
%        G       = code generator for the RSC code a in binary matrix
%        Lu      = extrinsic information from the previous decoder.
%        ind_dec= index of decoder=1/2 (assumed to be terminated/open)
% Output: L_A    = ln (P(x=1|y)/P(x=-1|y)), i.e., Log-Likelihood Ratio
%             (soft-value) of estimated message input bit at each level
lu=length(Ly)/2; Infty=1e2; EPS=1e-50; % Number of input bits, etc
[N,L] = size(G);
Ns = 2^(L-1);  % Number of states in the trellis
Le1=-log(1+exp(Lu)); Le2=Lu+Le1; % ln(exp((u+1)/2*Lu)/(1+exp(Lu)))
% Set up the trellis
[nout, ns, pout, ps] = trellis(G);
% Initialization of Alpha and Beta
Alpha(1,2:Ns) = -Infty; % Eq.(9.4.40) (the initial all-zero state)
if ind_dec==1 % for decoder D1 with termination in all-zero state
  Beta(lu+1,2:Ns) = -Infty; % Eq.(9.4.41b) (the final all-zero state)
 else % for decoder D2 without termination
  Beta(lu+1,:) = -log(Ns)*ones(1,Ns);
end
% Compute gamma at every depth level (stage)
for k = 2:lu+1
   Lyk = Ly(k*2-[3 2]);  gam(:,:,k) = -Infty*ones(Ns,Ns);
   for s2 = 1:Ns % Eq.(9.4.42)
      gam(ps(s2,1),s2,k) = Lyk*[-1 pout(s2,2)].' +Le1(k-1);
      gam(ps(s2,2),s2,k) = Lyk*[+1 pout(s2,4)].' +Le2(k-1);
   end
end
% Compute Alpha in forward recursion
for k = 2:lu
  for s2 = 1:Ns
    alpha = sum(exp(gam(:,s2,k).'+Alpha(k-1,:))); % Eq.(9.4.40)
    if alpha<EPS, Alpha(k,s2)=-Infty; else Alpha(k,s2)=log(alpha); end
  end
  tempmax(k) = max(Alpha(k,:)); Alpha(k,:) = Alpha(k,:)-tempmax(k);
end
% Compute Beta in backward recursion
for k = lu:-1:2
   for s1 = 1:Ns
      beta = sum(exp(gam(s1,:,k+1)+Beta(k+1,:))); % Eq.(9.4.41)
      if beta<EPS, Beta(k,s1)=-Infty; else Beta(k,s1)=log(beta); end
   end
   Beta(k,:) = Beta(k,:) - tempmax(k);
end
% Compute the soft output LLR for the estimated message input
for k = 1:lu
  for s2 = 1:Ns % Eq.(9.4.39)
    temp1(s2)=exp(gam(ps(s2,1),s2,k+1)+Alpha(k,ps(s2,1))+Beta(k+1,s2));
    temp2(s2)=exp(gam(ps(s2,2),s2,k+1)+Alpha(k,ps(s2,2))+Beta(k+1,s2));
  end
  L_A(k) = log(sum(temp2)+EPS) - log(sum(temp1)+EPS); % Eq.(9.4.38)
end
```

```
function [nout,nstate,pout,pstate] = trellis(G)
% copyright 1998, Yufei Wu, MPRG lab, Virginia Tech for academic use
% set up the trellis with code generator G in binary matrix form.
% G: Generator matrix with feedback/feedforward connection in row 1/2
% e.g. G=[1 1 1; 1 0 1] for the turbo encoder in Fig. 9.15(a)
% nout(i,1:2): Next output [xs=m xp](-1/+1) for state=i, message in=0
% nout(i,3:4): next output [xs=m xp](-1/+1) for state=i, message in=1
% nstate(i,1): next state(1,...2^M) for state=i, message input=0
% nstate(i,2): next state(1,...2^M) for state=i, message input=1
% pout(i,1:2): previous out [xs=m xp](-1/+1) for state=i, message in=0
% pout(i,3:4): previous out [xs=m xp](-1/+1) for state=i, message in=1
% pstate(i,1): previous state having come to state i with message in=0
% pstate(i,2): previous state having come to state i with message in=1
% See Fig. 9.16 for the meanings of the output arguments.
[N,L] = size(G); % Number of output bits and Consraint length
M=L-1;  Ns=2^M; % Number of bits per state and Number of states
% Set up next_out and next_state matrices for RSC code generator G
for state_i=1:Ns
   state_b = deci2bin1(state_i-1,M); % Binary state
   for input_bit=0:1
      d_k = input_bit;
      a_k=rem(G(1,:)*[d_k  state_b]',2); % Feedback in Fig.9.15(a)
      out(input_bit+1,:)=[d_k  rem(G(2,:)*[a_k  state_b]',2)]; % Forward
      state(input_bit+1,:)=[a_k  state_b(1:M-1)]; % Shift register
   end
   nout(state_i,:) = 2*[out(1,:) out(2,:)]-1; % bipolarize
   nstate(state_i,:) = [bin2deci(state(1,:)) bin2deci(state(2,:))]+1;
end
% Possible previous states having reached the present state
%  with input_bit=0/1
for input_bit=0:1
   bN = input_bit*N; b1 = input_bit+1; % Number of output bits = 2;
   for state_i=1:Ns
      pstate(nstate(state_i,b1),b1) = state_i;
      pout(nstate(state_i,b1),bN+[1:N]) = nout(state_i,bN+[1:N]);
   end
end
```

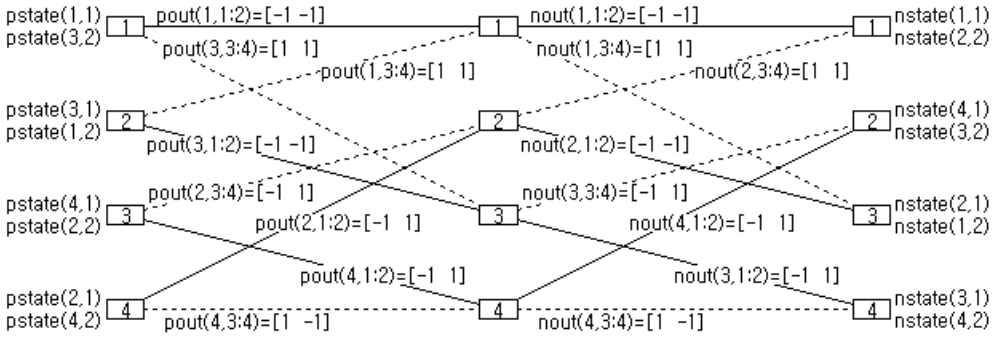


Figure 9.16 A trellis built by the routine 'trellis()' and its output arguments

<**SOVA (Soft-In/Soft-Output Viterbi Algorithm) Decoding cast into 'sova()''**> [H-2]

The objective of the SOVA-MAP decoding algorithm is to find the state sequence $\mathbf{s}^{(i)}$ and the corresponding input sequence $\mathbf{u}^{(i)}$ which maximizes the following MAP (maximum a posteriori probability) function:

$$P(\mathbf{s}^{(i)}|\,\mathbf{y}) \overset{(2.1.4)}{=} p(\mathbf{y}|\,\mathbf{s}^{(i)})\frac{P(\mathbf{s}^{(i)})}{p(\mathbf{y})} \overset{\text{proportional}}{\sim} p(\mathbf{y}|\,\mathbf{s}^{(i)})P(\mathbf{s}^{(i)}) \text{ for given } \mathbf{y} \tag{9.4.43}$$

This probability would be found from the multiplications of the branch transition probabilities defined by Eq. (9.4.42). However, as is done in the routine 'logmap()', we will compute the path meric by accumulating the logarithm or exponent of only the terms affected by $u_k^{(i)}$ as follows:

$$M_k(\mathbf{s}^{(i)}) = M_{k-1}(\mathbf{s}'^{(i)}) + \frac{L(u)}{2}u_k^{(i)} + \frac{1}{2}L_c[y_k^s \quad y_k^p]\begin{bmatrix} u_k^{(i)} \\ x_k^p(u_k^{(i)}) \end{bmatrix} \tag{9.4.44}$$

The decoding algorithm cast into the routine 'sova(Ly,G,Lu,ind_dec)' proceeds as follows:

(Step 0) Find the number of the $[y_k^s \;\; y_k^p]$'s in Ly given as the first input argument: l_u =length(Ly)/2. Find the number N of output bits of the two encoders and the constraint length L from the row and column dimensions of the generator matrix G. Let the number of states be $N_s = 2^{L-1}$, the SOVA window size δ=30, and the depth level $k=0$. Under the assumption of all-zero state at the initial stage (depth level zero), initialize the path metric to $M_k(s_0) = 0 = \ln 1$ (corresponding to probability 1) only for the all-zero state s_0 and to $M_k(s_j) = -\infty = \ln 0$ (corresponding to probability 0) for the other states s_j ($j \neq 0$).

(Step 1) Increment k by one and determine which one of the hypothetical encoder input (message) $u_{k-1}=0$ or $u_{k-1}=1$ would result in larger path metric $M_k(s_i)$ (computed by Eq. (9.4.44)) for every state $s_i (i=0:N_s-1)$ at level k and chooses the corresponding path as the survivor path, storing the estimated value of u_{k-1} (into 'pinput(i,k)') and the relative path metric difference 'DM(i,k)' of the survivor path over the other (non-surviving) path

$$\Delta M_k(s_i) = M_k(s_i \mid u_{k-1}=0/1) - M_k(s_i \mid u_{k-1}=1/0) \tag{9.4.45}$$

for every state at the stage. Repeat this step (in the forward direction) till $k=l_u$.

(Step 2) Depending on the value of the fourth input argument 'ind_dec', determine the all-zero state s_0 or any state belonging to the most likely path (with Max $M_k(s_i)$) to be the final state $\hat{s}(k)$ (sh(k)).

(Step 3) Find $\hat{u}(k)$ (uhat(k)) from 'pinput(i,k)' (constructed at Step 1) and the corresponding previous state $\hat{s}(k-1)$ (shat(k-1)) from the trellis structure. Decrement k by one. Repeat this step (in the backward direction) till $k=0$.

(Step 4) To find the reliability of $\hat{u}(k)$, let LLR= $\Delta M_k(\hat{s}(k))$. Trace back the non-surviving paths from the optimal states $\hat{s}(k+i)$ (for $i=1:\delta$ such that $k+i \le l_u$), find the nearly-optimal input $\hat{u}_i(k)$. If $\hat{u}_i(k) \neq \hat{u}(k)$ for some i, let LLR=Min{LLR, $\Delta M_{k+i}(\hat{s}(k+i))$}. In this way, find the LLR estimate and multiply it with the bipolarized value of $\hat{u}(k)$ to determine the soft output or L-value:

$$L_A(\hat{u}(k)) = (2\hat{u}(k)-1)\,\text{LLR} \tag{9.4.46}$$

```
function L_A = sova(Ly,G,Lu,ind_dec)
% Copyright: Yufei Wu, 1998, MPRG lab, Virginia Tech for academic use
% This implements Soft Output Viterbi Algorithm in trace back mode
% Input: Ly      : Scaled received bits Ly=0.5*L_c*y=(2*a*rate*Eb/N0)*y
%        G       : Code generator for the RSC code in binary matrix form
%        Lu      : Extrinsic information from the previous decoder.
%        ind_dec: Index of decoder=1/2
%                  (assumed to be terminated in all-zero state/open)
% Output: L_A   : Log-Likelihood Ratio (soft-value) of
%                   estimated message input bit u(k) at each stage,
%                 ln (P(u(k)=1|y)/P(u(k)=-1|y))
lu = length(Ly)/2; % Number of y=[ys yp] in Ly
lu1 = lu+1; Infty = 1e2;
[N,L] = size(G); Ns = 2^(L-1); % Number of states
delta = 30;  % SOVA window size
%Make decision after 'delta' delay. Tracing back from (k+delta) to k,
% decide bit k when received bits for bit (k+delta) are processed.
% Set up the trellis defined by G.
[nout,ns,pout,ps] = trellis(G);
% Initialize the path metrics to -Infty
Mk(1:Ns,1:lu1)=-Infty; Mk(1,1)=0; % Only initial all-0 state possible
% Trace forward to compute all the path metrics
for k=1:lu
  Lyk = Ly(k*2-[1 0]); k1=k+1;
  for s=1:Ns  % Eq.(9.4.44), Eq.(9.4.45)
    Mk0 = Lyk*pout(s,1:2).' -Lu(k)/2 +Mk(ps(s,1),k);
    Mk1 = Lyk*pout(s,3:4).' +Lu(k)/2 +Mk(ps(s,2),k);
    if Mk0>Mk1, Mk(s,k1)=Mk0; DM(s,k1)=Mk0-Mk1; pinput(s,k1)=0;
     else       Mk(s,k1)=Mk1; DM(s,k1)=Mk1-Mk0; pinput(s,k1)=1;
    end
  end
end
% Trace back from all-zero state or the most likely state for D1/D2
% to get input estimates uhat(k), and the most likely path (state) shat
if ind_dec==1, shat(lu1)=1;  else [Max,shat(lu1)]=max(Mk(:,lu1));  end
for k=lu:-1:1
   uhat(k)=pinput(shat(k+1),k+1); shat(k)=ps(shat(k+1),uhat(k)+1);
end
% As the soft-output, find the minimum DM over a competing path
%  with different information bit estimate.
for k=1:lu
   LLR = min(Infty,DM(shat(k+1),k+1));
   for i=1:delta
      if k+i<lu1
         u_=1-uhat(k+i); % the information bit
         tmp_state = ps(shat(k+i+1),u_+1);
         for j=i-1:-1:0
            pu=pinput(tmp_state,k+j+1); tmp_state=ps(temp_state,pu+1);
         end
         if pu~=uhat(k), LLR = min(LLR,DM(shat(k+i+1),k+i+1)); end
      end
   end
   L_A(k) = (2*uhat(k)-1)*LLR; % Eq.(9.4.46)
end
```

```
%turbo_code_demo.m
% simulates the classical turbo encoding-decoding system.
% 1st encoder is terminated with tails bits. (lm+M) bits are scrambled
% and passed to 2nd encoder, which is left open without termination.
clear
dec_alg = 1; % 0/1 for Log-MAP/SOVA
puncture = 1; % puncture or not
rate = 1/(3-puncture);  % Code rate
lu = 1000; % Frame size
Nframes = 100; % Number of frames
Niter = 4; % Number of iterations
EbN0dBs = 2.6; %[1 2 3];
N_EbN0dBs = length(EbN0dBs);
G = [1 1 1; 1 0 1]; % Code generator
a = 1; % Fading amplitude; a=1 in AWGN channel
[N,L]=size(G);  M=L-1;  lm=lu-M; % Length of message bit sequence
for nENDB = 1:N_EbN0dBs
   EbN0 = 10^(EbN0dBs(nENDB)/10); % convert Eb/N0[dB] to normal number
   L_c = 4*a*EbN0*rate;          % reliability value of the channel
   sigma = 1/sqrt(2*rate*EbN0); % standard deviation of AWGN noise
   noes(nENDB,:) = zeros(1,Niter);
  for nframe = 1:Nframes
      m = round(rand(1,lm));    % information message bits
      [temp,map] = sort(rand(1,lu));  % random interleaver mapping
      x = encoderm(m,G,map,puncture); % encoder output [x(+1/-1)
      noise = sigma*randn(1,lu*(3-puncture));
      r = a.*x + noise; % received bits
      y = demultiplex(r,map,puncture); % input for decoder 1 and 2
      Ly = 0.5*L_c*y; % Scale the received bits
      for iter = 1:Niter
         % Decoder 1
         if iter<2, Lu1=zeros(1,lu); % Initialize extrinsic information
          else Lu1(map)=L_e2; % (deinterleaved) a priori information
         end
         if dec_alg==0, L_A1=logmap(Ly(1,:),G,Lu1,1); % all information
          else          L_A1=sova(Ly(1,:),G,Lu1,1); % all information
         end
         L_e1= L_A1-2*Ly(1,1:2:2*lu)-Lu1; % Eq.(9.4.47)
         % Decoder 2
         Lu2 = L_e1(map); % (interleaved) a priori information
         if dec_alg==0, L_A2=logmap(Ly(2,:),G,Lu2,2); % all information
          else          L_A2=sova(Ly(2,:),G,Lu2,2); % all information
         end
         L_e2= L_A2-2*Ly(2,1:2:2*lu)-Lu2; % Eq.(9.4.47)
         mhat(map)=(sign(L_A2)+1)/2; % Estimate the message bits
         noe(iter)=sum(mhat(1:lu-M)~=m); % Number of bit errors
      end  % End of iter loop
      % Total number of bit errors for all iterations
      noes(nENDB,:) = noes(nENDB,:) + noe;
      ber(nENDB,:) = noes(nENDB,:)/nframe/(lu-M); % Bit error rate
      fprintf('\n');
      for i=1:Niter, fprintf('%14.4e ', ber(nENDB,i)); end
    end   % End of nframe loop
end   % End of nENDB loop
```

Now, it is time to take a look at the main program "turbo_code_demo.m", which uses the routine 'logmap()' or 'sova()' (corresponding to the block named 'Log-MAP or SOVA' in Fig. 9.15(c)) as well as the routines 'encoderm()' (corresponding to Fig. 9.15(a)), 'rsc_encode()', 'demultiplex()' (corresponding to Fig. 9.15(b)), and 'trellis()' to simulate the turbo coding system depicted in Fig. 9.15. All of the programs listed here in connection with turbo coding stem from the routines developed by Yufei Wu in the MPRG (Mobile/Portable Radio Research Group) of Virginia Tech. (Polytechnic Institute and State University). The following should be noted:

- One thing to note is that the *extrinsic information* L_e to be presented to one decoder i by the other decoder j should contain only the intrinsic information of decoder j that is obtained from its own parity bits not available to decoder i. Accordingly, one decoder should remove the information about y^s (avaiable commonly to both decoders) and the priori information $L(u)$ (provided by the other decoder) from the overall information L_A to produce the information that will be presented to the other decoder. (Would your friend be glad if you gave his/her present back to him/her or presented him/her what he/she had already got?) To prepare an equation for this information processing job of each encoder, we extract only the terms affected by $u_k = \pm 1$ from Eqs. (9.4.44) and (9.4.42) (each providing the basis for the path metric (Eq. (9.4.45)) and LLR (Eq. (9.4.38)), respectively,) to write

$$\left(\frac{L(u)}{2}u_k^{(i)}+\frac{1}{2}L_c y_k^s u_k^{(i)}\right)\Bigg|_{u_k^{(i)}=+1} - \left(\frac{L(u)}{2}u_k^{(i)}+\frac{1}{2}L_c y_k^s u_k^{(i)}\right)\Bigg|_{u_k^{(i)}=-1} = L(u) + L_c y_k^s$$

  which conforms with Eq. (9.4.37) for the conditioned LLR $L_{\mathbf{u}|\mathbf{y}}(u \mid y)$. To prepare the extrinsic information for the other decoder, this information should be removed from the overall information $L_A(u)$ produced by the the routine 'logmap()' or 'sova()' as

$$L_e(u) = L_A(u) - L(u) - L_c y_k^s \tag{9.4.47}$$

- Another thing to note is that as shown in Fig. 9.15(c), the basis for the final decision about u is the deinterleaved overall information L_{A2} that is attributed to decoder 2. Accordingly, the turbo decoder should know the pseudo-random sequence 'map' (that has been used for interleaving by the transmitter) as well as the fading amplitude and SNR of the channel.
- The trellis structure and the output arguments produced by the routine 'trellis()' are illustrated in Fig. 9.16.

Interest readers are invited to run the program "turbo_code_demo.m" with the value of the control constant 'dec_alg' set to 0/1 for Log-MAP/SOVA decoding algorithm and see the BER becoming lower as the decoding iteration proceeds. How do the turbo codes work? How are the two decoding algorithms, Log-MAP and SOVA, compared? Is there any weakpoint of turbo codes? What is the measure against the weakpoint, if any? Unfortunately, to answer such questions is difficult for the authors and therefore, is beyond the scope of this book. As can be seen from the simulation results, turbo codes have an excellent BER performance close to the Shannon limit at low and medium SNRs. However, the decreasing rate of the BER curve of a turbo code can be very low at high SNR depending on the interleaver and the free distance of the code, which is called the '*error floor*' phenomenon. Besides, turbo codes needs not only a large interleaver and block size but also many iterations to achieve such a good BER performance, which increases the complexity and latency (delay) of the decoder.

(d) Why don't we make the code for the above set of four symbols as follows?

Symbol	00	01	10	11
Codeword	0	1	10	11

With this code, we could have the average codeword length as

$$L = 0.81\times 1+0.09\times 1+0.09\times 2+0.01\times 2 = 1.1[\text{bits/symbol}]$$

which is shorter than that with the code obtained in (a) or (b). To answer this question, think about the decoding of a coded sequence {1011} based on the above table.

9.4 Channel Capacity and Hamming Codes

Recall that the MATLAB function 'Hammgen(n)' can be used to generate the $n\times N$ parity-check matrix for an (N,K) Hamming code where $N=2^n-1$ and $K=N-n$. Suppose the message sequence is coded with the (7,4) or (15,11) Hamming code, BPSK(Binary Phase Keying)-modulated, and then transmitted through a BSC (Binary Symmetric Channel).

(a) Compute the crossover (channel bit transmission error) probability of BPSK signaling with each of these two codings for SNRdB=0~13 [dB] by

$$\varepsilon \overset{(7.3.5)}{=} Q\left(\sqrt{SNR_r R_c}\right) \text{ with } R_c(\text{Code rate}) = \frac{K}{N} \tag{P9.4.1}$$

and substitute this into Eq. (9.3.13) to get the channel capacity

$$C_t \overset{(9.3.13)}{=} 1+(1-\varepsilon)\log_2(1-\varepsilon)+\varepsilon\log_2\varepsilon = 1-H_b(\varepsilon)\,[\text{bits/symbol}] \tag{P9.4.2}$$

Plot the channel capacity C_t vs. SNRdB=0~13 and use the graph to find the rough value of SNRdB at which the inequality (9.3.8) is marginally satisfied:

$$b_c = R_c b \underset{\text{BPSK}}{\overset{b=1}{=}} R_c \le C_t\,[\text{bits/symbol}] \tag{P9.4.3}$$

(b) Referring to the lower part of the MATLAB routine 'do_Hamming_code74()' (Sec. 9.4.2), use Eq. (9.4.11) to compute the theoretical (approximate) probabilities of message bit error for SNRdB=0~13 [dB] with no code, (7,4) Hamming code, and (15,11) Hamming code and plot them versus SNRdB. Use the theoretical BER curves to find the rough values of SNRdB at which the two codings have the same BER as that with no coding.

(c) Modify the MATLAB routine 'do_Hamming_code74()' so that it can simulate the BPSK signaling with (7,4) Hamming code and (15,11) Hamming code for SNRdB = 5, 7, 9, and 11[dB]. Run it to plot the BERs versus SNRdB on the theoretical BER curves obtained in (b).

Table P9.4 BERs of BPSK signaling with (7,4) or (15,11) Hamming code

	SNRdB=5[dB]	SNRdB=7[dB]	SNRdB=9[dB]	SNRdB=11[dB]
No code	0.037679		0.002413	
(7,4) Hamming code		0.012422 (0.010996)		0.000118 (0.000080)
(15,11) Hamming code	0.060606 (0.038549)		0.001196 (0.000830)	

9.5 Effect of Coding on BER

To understand why the BER of simulation result is much higher than the theoretical value of Eq. (9.4.11) (Sec. 9.4.2), insert the following statements at the appropriate places in the MATLAB routine 'do_Hamming_code74()' (Sec. 9.4.2).

```
notbe1= sum(r_sliced~=coded); notbec1= sum(r_c~=coded);
si= bin2deci(s); % syndrome
if notbe1>0 & iter<30
  fprintf('\n # of transmitted bit errors %d -> ',notbe1);
  fprintf('%d after correction (syndrome= %d)',notbec1,si);
end
```

Then run the routine with SNRbdB=5 to answer the following questions:

(i) Is a single bit error in a codeword always corrected?

(ii) Is there any case where the number of bit errors is increased by a wrong correction?

What do you think is the reason for the big gap between the two BERs that are obtained from simulation and theoretical formula (9.4.11)?

9.6 BCH (Bose-Chaudhuri-Hocquenghem) Codes with Simulink

BCH codes constitute a powerful class of cyclic codes that provides a large selection of block lengths, code rates, alphabet sizes, and error-correcting capability as listed in Table 5.2 of [S-3]. Accroding to the table, the $(N, K) = (15,7)$ and (31,16) BCH codes are represented by the generator polynomial coefficient vectors

$$\mathbf{g}_1 = 721(\text{octal}) = [1 \;\; 1 \;\; 1 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 1] \tag{P9.6.1}$$

$$\mathbf{g}_2 = 107657(\text{octal}) = [1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 0 \;\; 1 \;\; 0 \;\; 1 \;\; 1 \;\; 1 \;\; 1] \tag{P9.6.2}$$

and their error correcting capabilities are 2 and 3, rspectively. Suppose the message sequence is coded with these two BCH codes, BPSK(binary phase-shift keying)-modulated, and then transmitted through a BSC (binary symmetric channel).

(a) Let us try to make use of the MATLAB built-in function 'bchgenpoly()' to make the generator polynomial vectors for the (31,16) BCH code:

```
N=31; K=16;
gBCH=bchgenpoly(N,K); % Galois row vector in GF(2) representing
                      % the (N,K) BCH code
g=double(gBCH.x) % Extracting the elements from a Galois array
```

Does the resulting vector conform with $\mathbf{g}_2$?

(b) It may take a long time to use the MATLAB built-in routine 'decode()' with such a big BCH code as $\mathbf{g}_2$. If we prepare the syndrome table using 'E=syndtable(H)' beforehand and then use 'decode(coded,N,K,'cyclic',g,E)' with the prepared syndrome table E, it will save a lot of decoding time. Likewise, if we prepare the error pattern matrix E and the corresponding error pattern index vector epi using 'cyclic_decoder0' (Sec. 9.4.3) beforehand and then use 'cyclic_decoder(coded,N,K,g,E,epi)', it will save the decoding time, too. Complete the following MATLAB program "do_cyclic_codes.m" so that it can simulate a BPSK communication system with each of the above two BCH codes and plot the BERs for SNRdBs1=[2 4 10] together with the throretical BER curves for SNRdBs=[0:0.01:14] obtained from Eq. (9.4.11). Run the completed program to get the BER curves.

```
%do_cyclic_codes.m
clear, clf
gsymbols=['bo';'r+';'kx';'md';'g*'];
% Theoretical BER curves
SNRdBs=0:0.01:14;  SNRs=10.^(SNRdBs/10); pemb_uncoded=Q(sqrt(SNRs));
semilogy(SNRdBs,pemb_uncoded,':'), hold on
use_decode = 0; % Use encode()/decode or not
MaxIter=1e5; Target_no_of_error=100;  SNRdBs1=2:4:10;
for iter=1:2
   if iter==1, N=15; K=7; nceb=2; %(15,7) BCH code generator
    else    N=31; K=16; nceb=3; % (31,16) BCH code generator
   end
   gBCH=bchgenpoly(N,K); %Galois row vector representing (N,K) BCH code
   g=double(gBCH.x);
    clear('S')
   if use_decode>0, H=cyclgen(N,g); E=syndtable(?);
    else
     E= combis(N,nceb); % All error patterns
     for i=1:size(E,1)
        S(i,:) = cyclic_decoder0(E(i,:),N,K,g); % Syndrome
        epi(bin2deci(S(i,:))) = ?; % Error pattern indices
     end
   end
   Rc = K/N;  %code rate
   SNRcs=SNRs*Rc; sqrtSNRcs=sqrt(SNRcs);
   ets=Q(sqrt(SNRcs)); % transmitted bit error probability Eq.(7.3.5)
   pemb_t=prob_err_msg_bit(ets,N,nceb);
    semilogy(SNRdBs,pemb_t,gsymbols(iter,1),'Markersize',5)
    for iter1=1:length(SNRdBs1)
       SNRdB = SNRdBs1(iter1); SNR = 10.^(SNRdB/10);
       SNRc = SNR*Rc; sqrtSNRc = sqrt(SNRc);
       et=Q(sqrt(SNRc)); % transmitted bit error probability Eq.(7.3.5)
       K100 = K*100;
       nombe = 0;
       for iter2=1:MaxIter
          msg=randint(1,K100); % Message vector
          if use_decode==0,  coded=cyclic_???????(msg,N,K,g);
           else  msg=reshape(msg,length(msg)/K,K);
                 coded=??????(msg,N,K,'cyclic',g);
          end
          r= 2*coded-1 + randn(size(coded))/sqrtSNRc; % Received vector
          r_sliced= 1*(r>0); % Sliced
          if use_decode==0
            decoded= cyclic_???????(r_sliced,N,K,g,E,epi);
           else  decoded= ??????(r_sliced,N,K,'cyclic',g,E);
          end
          nombe= nombe + sum(sum(decoded~=msg));
          if nombe>Target_no_of_error, break; end
       end
       lm=iter2*K100; pemb=nombe/lm; % Message bit error probability
       semilogy(SNRdB,pemb,gsymbols(iter,:),'Markersize',5)
   end
end
```

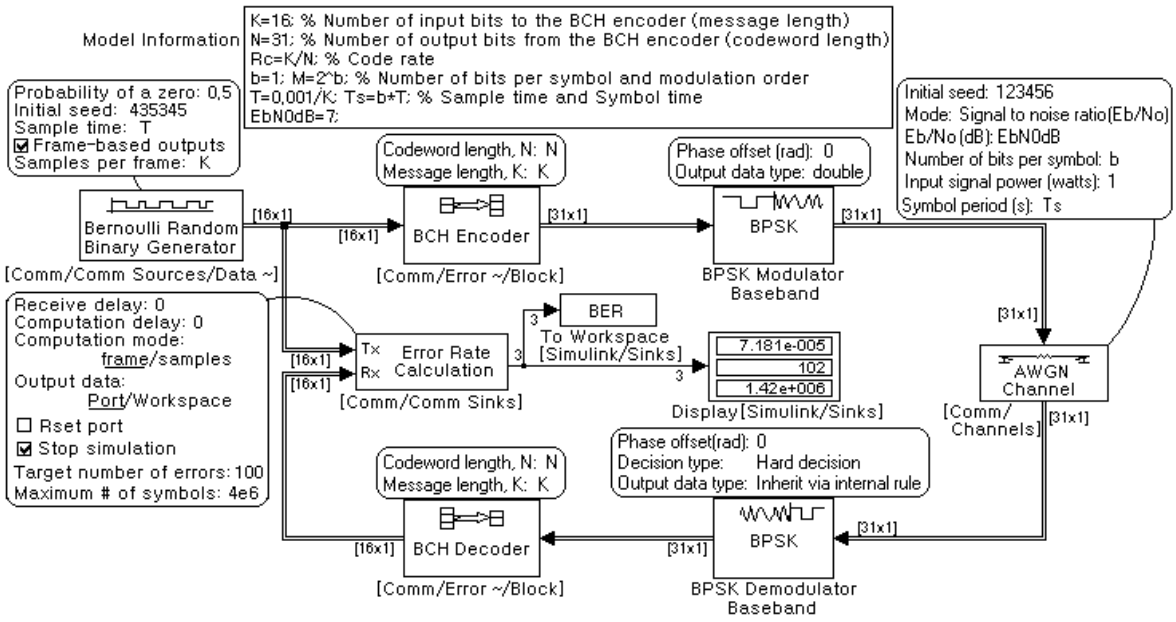
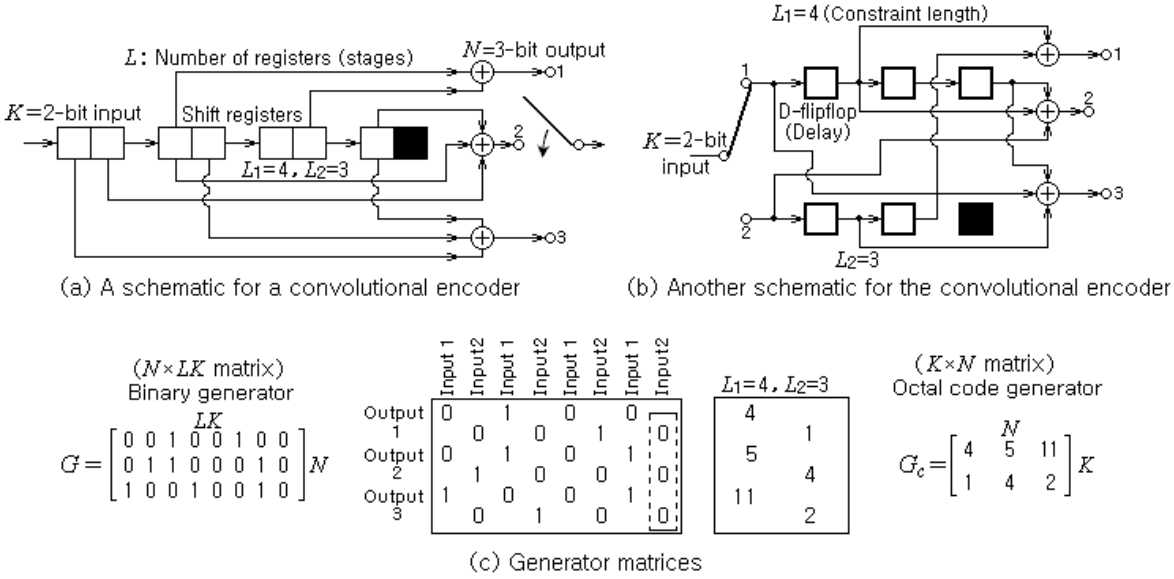


Figure P9.6 Simulink model "BCH_BPSK_sim.mdl"

(c) Referring to the Simulink model "BCH_BPSK_sim.mdl" depicted in Fig. P9.6 and the following program "do_BCH_BPSK_sim.m", use Simulink to simulate a BPSK communication system with the (31,16) BCH code for error correction and find the BERs for $E_b/N_0 = -1$, 3, and 7dB. Noting that SNRbdB=EbN0dB+3 since $SNR_b = E_b/(N_0/2)$, list the BERs together with those obtained using 'encode()'/'decode()' in (b).

```
%do_BCH_BPSK_sim.m
clear, clf
K=16; % Number of input bits to the BCH encoder (message length)
N=31; % Number of output bits from the BCH encoder (codeword length)
Rc=K/N; % Code rate to be multiplied with the SNR in AWGN channel block
b=1; M=2^b; % Number of bits per symbol and modulation order
T=0.001/K; Ts=b*T; % Sample time and Symbol time
SNRbdBs=[2:4:10]; EbN0dBs=SNRbdBs-3;
EbN0dBs_t=0:0.1:10; EbN0s_t=10.^(EbN0dBs_t/10);
SNRbdBs_t=EbN0dBs_t+3;
BER_theory= prob_error(SNRbdBs_t,'PSK',b,'BER');
for i=1:length(EbN0dBs)
   EbN0dB=EbN0dBs(i);
   sim('BCH_BPSK_sim');
   BERs(i)=BER(1); % just ber among {ber, # of errors, total # of bits}
   fprintf(' With EbN0dB=%4.1f, BER=%10.4e=%d/%d\n', EbN0dB,BER);
end
semilogy(EbN0dBs,BERs,'r*', EbN0dBs_t,BER_theory,'b')
xlabel('Eb/N0[dB]'); ylabel('BER');
title('BER of BCH code with BPSK');
```



(a) A schematic for a convolutional encoder

(b) Another schematic for the convolutional encoder

(c) Generator matrices

Figure P9.7 A convolutional encoder and its generator matrix

```
%dc09p07.m
% To practice using convenc() and vitdec() for channel coding
clear, clf
Gc=[4 5 11;1 4 2]; % Octal code generator matrix
K=size(Gc,1); % Number of encoder input bits
% Constraint length vector
Gc_m=max(Gc.');
for i=1:length(Gc_m), Lc(i)=length(deci2bin1(oct2dec(Gc_m(i))));  end
trel=poly2trellis(Lc,Gc);
Tbdepth=sum(Lc)*5; delay=Tbdepth*K;
lm=1e5; msg=randint(1,lm);
transmission_ber=0.02;
notbe=round(transmission_ber*lm); % Number of transmitted bit errors
ch_input=convenc([msg zeros(1,delay)],trel);
% Received/modulated/detected signal
ch_output= rem(randerr(1,length(ch_input),notbe)+ch_input,2);
decoded_trunc= vitdec(ch_output,trel,Tbdepth,'trunc','hard');
ber_trunc= sum(msg~=decoded_trunc(????))/lm;
decoded_cont= vitdec(ch_output,trel,Tbdepth,'cont','hard');
ber_cont=sum(msg~=decoded_cont(???????????))/lm;
% It is indispensable to use the delay for the decoding result
%  obtained using vitdec(,,,'cont',)
nn=[0:100-1];
subplot(221), stem(nn,msg(nn+1)), title('Message sequence')
subplot(223), stem(nn,decoded_cont(nn+1)), hold on
stem(delay,0,'rx')
decoded_term= vitdec(ch_output,trel,Tbdepth,'term','hard');
ber_term=sum(msg~=decoded_term(????))/lm;
fprintf('\n BER_trunc  BER_cont   BER_term')
fprintf('\n %9.2e  %9.2e  %9.2e\n', ber_trunc,ber_cont,ber_term)
```

9.7 A Convolutional Code and Viterbi Decoding

Fig. P9.7 shows a convolutional encoder (described by two kinds of schematic) and the two corresponding generator matrices where one is a binary generator matrix that is used by the 'conv_encoder()'/'conv_decoder()' and the other is an octal generator matrix that is used by the MATLAB built-in functions 'convenc()'/'vitdec()'. Let the convolutional encoder and the corresponding Viterbi decoder be used for channel coding where there happens a 2%-error in the 100,000 transmitted bits. Complete the above program "dc09p07.m" to simulate this situation and run it to find the BER.

9.8 A Convolutional Code and Viterbi Decoding with Simulink

Fig. P9.8.1 shows a convolutional encoder described by the octal code generator matrix $G_c = [133\ 171]$. Fig. P9.8.2 shows the Simulink model "Viterbi_QAM_sim.mdl" that can be used to simulate a QAM communication with a given convolutional encoder and the corresponding Viterbi decoder. The following MATLAB function 'Viterbi_QAM()' can also be used to simulate a QAM communication with a given convolutional encoder and the corresponding Viterbi decoder. Complete the following MATLAB program "do_Viterbi_QAM", which uses the MATLAB function 'Viterbi_QAM()' and the Simulink model "Viterbi_QAM_sim.mdl" to simulate a 16-QAM communication system with the convolutional encoder of Fig. P9.8.1 and the corresponding Viterbi decoder. Run it to find the BERs for EbN0=3, 6, and 9dB.

	EbN0dB=3[dB]	EbN0dB=6[dB]	EbN0dB=9[dB]
MATLAB 'Viterbi_QAM.m'		0.006684	
Simulink 'Viterbi_QAM_sim.mdl'	0.466403		0.000079

```
function [pemb,nombe,notmb]=Viterbi_QAM(Gc,b,SNRbdB,MaxIter)
if nargin<4, MaxIter=1e5; end
if nargin<3, SNRbdB=5; end
if nargin<2, b=4; end
[K,N]=size(Gc); Rc=K/N;  Gc_m=max(Gc.');
% Constraint length vector
for i=1:length(Gc_m), Lc(i)=length(deci2bin1(oct2dec(Gc_m(i)))); end
Nf=144; % Number of bits per frame
Nmod=Nf*N/K/b; % Number of QAM symbols per modulated frame
SNRb=10.^(SNRbdB/10); SNRbc=SNRb*Rc; sqrtSNRbc=sqrt(SNRbc);
sqrtSNRc=sqrt(2*b*SNRbc); % Complex noise for b-bit (coded) symbol
trel=poly2trellis(Lc,Gc);
Tbdepth=5; delay=Tbdepth*K;
nombe=0; Target_no_of_error=100;
for iter=1:MaxIter
   msg=randint(1,Nf); % Message vector
   coded= convenc(msg,trel); % Convolutional encoding
   modulated= QAM(coded,b); % 2^b-QAM-Modulation
   r= modulated +(randn(1,Nmod)+j*randn(1,Nmod))/sqrtSNRc;
   demodulated= QAM_dem(r,b); % 2^b-QAM-Demodulation
   decoded= vitdec(demodulated,trel,Tbdepth,'trunc','hard');
   nombe = nombe + sum(msg~=decoded(1:Nf));  %
  if nombe>Target_no_of_error, break; end
end
notmb=Nf*iter; % Number of total message bits
pemb=nombe/notmb; % Message bit error probability
```

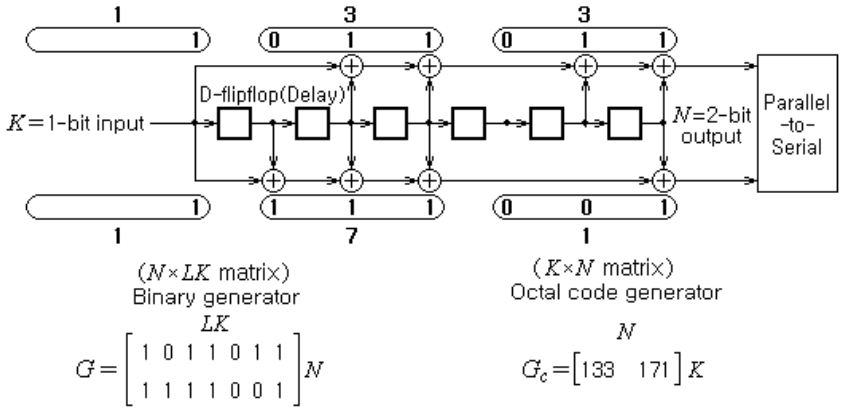


Figure P9.8.1 A convolutional encoder and its generator matrix

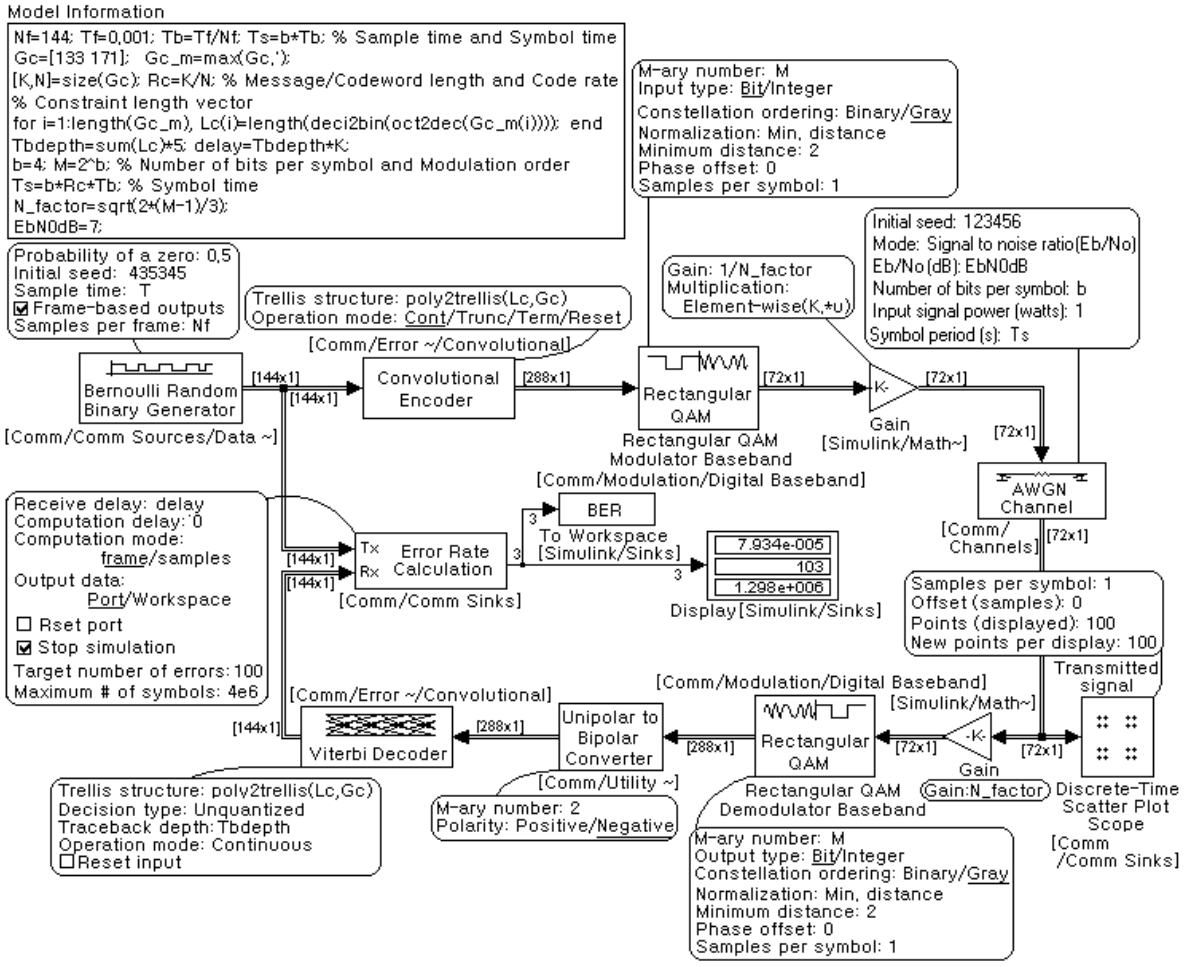


Figure P9.8.2 Simulink model "Viterbi_QAM_sim.mdl"

```
%do_Viterbi_QAM.m
clear, clf
Nf=144; Tf=0.001; Tb=Tf/Nf; % Frame size, Frame and Sample/Bit time
Gc=[133 171];
[K,N]=size(Gc); Rc=K/N; % Message/Codeword length and Code rate
% Constraint length vector
Gc_m=max(Gc.');
for i=1:length(Gc_m)
   Lc(i)=length(deci2bin1(oct2dec(Gc_m(i))));
end
Tbdepth=sum(Lc)*5; delay=Tbdepth*K;
b=4; M=2^b; % Number of bits per symbol and Modulation order
Ts=b*Rc*Tb; % Symbol time
N_factor=sqrt(2*(M-1)/3); % Eq.(7.5.4a)
EbN0dBs_t=0:0.1:10; SNRbdBs_t=EbN0dBs_t+3;
BER_theory= prob_error(SNRbdBs_t,'QAM',b,'BER');
EbN0dBs=[3 6]; Target_no_of_error=50;
for i=1:length(EbN0dBs)
   EbN0dB=EbN0dBs(i); SNRbdB=EbN0dB+3;
   randn('state', 0);
   [pemb,nombe,notmb]=???????_QAM(Gc,b,SNRbdB,Target_no_of_error);
   pembs(i)=pemb;
   sim('Viterbi_QAM_sim'); pembs_sim(i)=BER(1);
end
[pembs; pembs_sim]
semilogy(EbN0dBs,pembs,'r*', EbN0dBs_t,BER_theory,'b')
xlabel('Eb/N0[dB]'); ylabel('BER');
```

```
function qamseq=QAM(bitseq,b)
bpsym = nextpow2(max(bitseq)); % no of bits per symbol
if bpsym>0, bitseq = deci2bin(bitseq,bpsym);  end
if b==1, qamseq=bitseq*2-1; return; end  % BPSK modulation
% 2^b-QAM modulation
N0=length(bitseq);  N=ceil(N0/b);
bitseq=bitseq(:).'; bitseq=[bitseq zeros(1,N*b-N0)];
b1=ceil(b/2); b2=b-b1; b21=b^2; b12=2^b1; b22=2^b2;
g_code1=2*gray_code(b1)-b12+1; g_code2=2*gray_code(b2)-b22+1;
tmp1=sum([1:2:2^b1-1].^2)*b21; tmp2=sum([1:2:b22-1].^2)*b12;
M=2^b; Kmod=sqrt(2*(M-1)/3);
%Kmod=sqrt((tmp1+tmp2)/2/(2^b/4)) % Normalization factor
qamseq=[];
for i=0:N-1
   bi=b*i; i_real=bin2deci(bitseq(bi+[1:b1]))+1;
   i_imag=bin2deci(bitseq(bi+[b1+1:b]))+1;
   qamseq=[qamseq (g_code1(i_real)+j*g_code2(i_imag))/Kmod];
end
```

```
function [g_code,b_code]=gray_code(b)
N=2^b; g_code=0:N-1;
if b>1, g_code=gray_code0(g_code); end
b_code=deci2bin(g_code);
```

```
function g_code=gray_code0(g_code)
N=length(g_code); N2=N/2;
if N>=4, N2=N/2; g_code(N2+1:N)=fftshift(g_code(N2+1:N)); end
if N>4, g_code=[gray_code0(g_code(1:N2))
gray_code0(g_code(N2+1:N))]; end
```

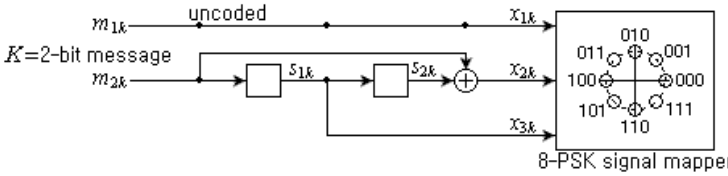
```
function bitseq=QAM_dem(qamseq,b,bpsym)
%BPSK demodulation
if b==1, bitseq=(qamseq>=0); return; end
%2^b-QAM demodulation
N=length(qamseq);
b1=ceil(b/2); b2=b-b1;
g_code1=2*gray_code(b1)-2^b1+1; g_code2=2*gray_code(b2)-2^b2+1;
tmp1=sum([1:2:2^b1-1].^2)*2^b2;
tmp2=sum([1:2:2^b2-1].^2)*2^b1;
Kmod=sqrt((tmp1+tmp2)/2/(2^b/4));    % Normalization factor
g_code1=g_code1/Kmod; g_code2=g_code2/Kmod;
bitseq=[];
for i=1:N
  [emin1,i1]=min(abs(real(qamseq(i))-g_code1));
  [emin2,i2]=min(abs(imag(qamseq(i))-g_code2));
  bitseq=[bitseq deci2bin1(i1-1,b1) deci2bin1(i2-1,b2)];
end
if (nargin>2)
  N = length(bitseq)/bpsym; bitmatrix = reshape(bitseq,bpsym,N).';
  for i=1:N, intseq(i)=bin2deci(bitmatrix(i,:));  end
  bitseq = intseq;
end
```

9.9 Ungerboeck TCM (Trellis-Coded Modulation) Codes

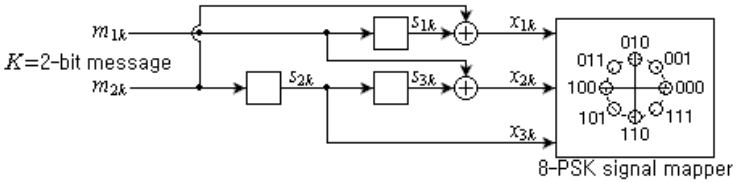
Figs. P9.9.1(a) and (b) show two Ungerboeck TCM encoders and Fig. P9.9.2 shows a Simulink model "TCM_sim.mdl" that simulates the TCM encoding-decoding with the TCM encoder of Fig. P9.9.1.

```
function [pemb,nombe,notmb]=...
      TCM(state_eq,K,Nsb,N,Constellation,SNRbdB,Target_no_of_error)
M=2^N; Rc=K/N;
Nf=288; % Number of bits per frame
Nmod=Nf/K; % Number of symbols per modulated frame
SNRb=10.^(SNRbdB/10); SNRbc=SNRb*Rc;
sqrtSNRc=sqrt(2*N*SNRbc); % Complex noise per K=Rc*N-bit symbol
nombe=0;  MaxIter=1e6;
for iter=1:MaxIter
   msg=randint(1,Nf); % Message vector
   coded = TCM_encoder(state_eq,K,Nsb,N,msg,Constellation);
   r= coded +(randn(1,Nmod)+j*randn(1,Nmod))/sqrtSNRc;
   decoded= TCM_decoder(state_eq,K,Nsb,r,Constellation);
   nombe = nombe + sum(msg~=decoded(1:Nf));
   if nombe>Target_no_of_error, break; end
end
notmb=Nf*iter; % Number of total message bits
pemb=nombe/notmb; % Message bit error probability
```

```
function [s1,x]=TCM_state_eq1(s,u,Constellation)
% State equation for the TCM encoder in Fig. P9.9.1(a)
% Input:  s= State, u= Input
% Output: s1= Next state, x= Output
s1 = [u(2) s(1)];
x= Constellation(bin2deci([u(1) rem(u(2)+s(2),2) s(1)])+1);
```



(a) A Ungerboeck TCM encoder consisting of 2^2-state FSM and 8-PSK signal mapper



(b) A Ungerboeck TCM encoder consisting of 2^3-state FSM and 8-PSK signal mapper

Figure P9.9.1 Ungerboeck TCM encoder

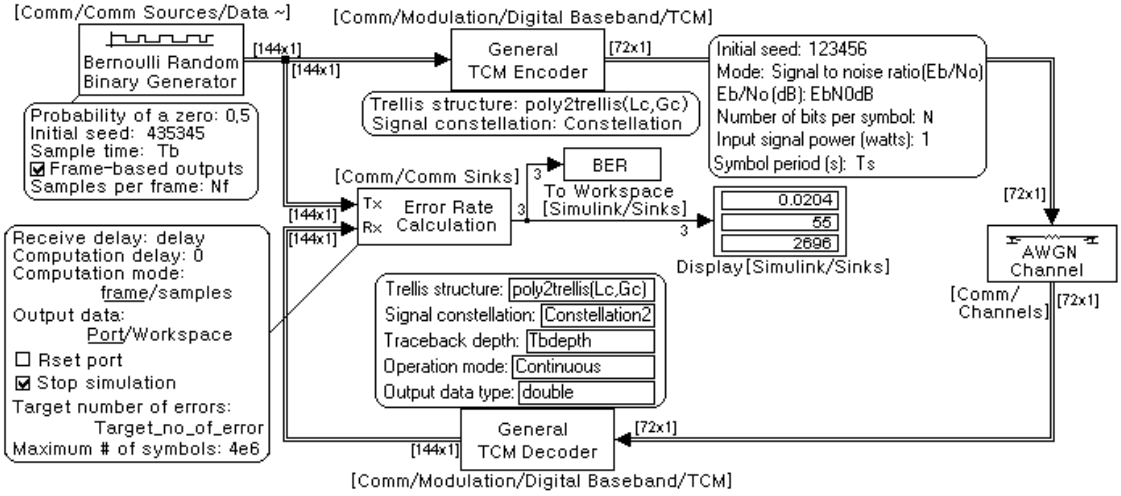


Figure P9.9.2 Simulink model for the Ungerboeck TCM of Figure P9.9.1 ("TCM_sim.mdl")

(a) Run the following program "do_TCM_8PSK.m" to use the above MATLAB routine "TCM()" and the Simulink model "TCM_sim.mdl" for finding the BERs of the TCM system with the encoder-modulator of Fig. P9.9.1(a) where the SNR is given as $E_b/N_0 = 2[\text{dB}]$. Set the 8PSK constellation for "TCM_sim.mdl" to C_1 or C_3 (Gray code).

$$
\begin{aligned}
C_1 &= \left\{ \exp\left(j\frac{2k\pi}{8} \right),\ k=0,1,2,3,4,5,6,7 \right\} \\
C_2 &= \left\{ \exp\left(j\frac{2k\pi}{8} \right),\ k=0,4,2,6,1,5,3,7 \right\} \\
C_3 &= \left\{ \exp\left(j\frac{2k\pi}{8} \right),\ k=0,1,3,2,6,7,5,4 \right\}
\end{aligned}
\tag{P9.9.1}
$$

What difference will be made to the BER? Set the 8PSK constellation for "TCM.m" to C_2 or C_3. What difference will be made to the BER? Fill in the following table.

Table P9.9 BER of TCM systems

E_b/N_0	BER			
	QPSK - no code (Theoretical)		TCM-8PSK (Fig. P9.9.1(a))	TCM-8PSK (Fig. P9.9.1(b))
2dB	0.038	TCM with C_1	0.0018	
		TCM1 with C_1		0.0020
		TCM with C_2	0.059	
		TCM1 with C_2		0.016
		TCM with C_3	0.018	
		TCM1 with C_3		0.015
		Simulink with C_2	0.020	
		Simulink with C_1		0.049
		Simulink with C_3	0.064	

(b) Modify the program "do_TCM_8PSK.m" and make a routine "TCM_state_eq2()" so that the TCM system with the encoder of Fig. P9.9.1(b) can be simulated. Run the modified program "do_TCM_8PSK.m" to find the BERs for $E_b/N_0 = 2$[dB] and fill in the corresponding blanks of Table P9.9.

```
%do_TCM_8PSK.m
clear
Nf=144; Tf=0.001; Tb=Tf/Nf; % Frame size, Frame and Sample/Bit times
Gc=[1 0 0; 0 5 2]; state_eq='TCM_state_eq1'; Ns=2; % Fig. P9.9.1(a)
%Gc=[1 2 0; 4 1 2]; state_eq='TCM_state_eq2'; Ns=3; % Fig. P9.9.1(b)
[K,N]=size(Gc); Rc=K/N; % Message/Codeword length and Code rate
% Constellation for TCM Simuklink block
Constellation2=exp(j*2*pi/8*[0 4 2 6 1 5 3 7]);
% Constellation good for TCM() MATLAB function
Constellation1=exp(j*2*pi/8*[0:7]);
% Constraint length vector
Gc_m=max(Gc.');
for i=1:length(Gc_m), Lc(i)=length(deci2bin(oct2dec(Gc_m(i)))); end
%trel=poly2trellis(Lc,Gc);
Tbdepth=sum(Lc)*5; delay=Tbdepth*K;
Ts=K*Tb; % or N*Rc*Tb % Symbol time
EbN0dBs_t=0:0.1:10;  SNRbdBs_t=EbN0dBs_t+3;
BER_theory= prob_error(SNRbdBs_t,'PSK',K,'BER');
EbN0dBs=2; %[3 6];
Target_no_of_error=50;
for i=1:length(EbN0dBs)
   EbN0dB=EbN0dBs(i); SNRbdB=EbN0dB+3;
   pemb=TCM(state_eq,K,Ns,N,Constellation1,SNRbdB,Target_no_of_error);
   pembs_TCM8PSK(i)=pemb
   sim('TCM_sim'); pembs_TCM8PSK_sim(i)=BER(1);
end
figure(1), clf
semilogy(EbN0dBs_t,BER_theory,'b'), hold on
semilogy(EbN0dBs,pembs_TCM8PSK_sim,'r*', EbN0dBs,pembs_TCM8PSK,'m*')
```

(c) For the TCM encoder with no input bit inserted into between the flip-flops (registers), the routines 'TCM()', 'TCM_encoder(), and 'TCM_decoder()' can be modified as 'TCM1()', 'TCM_encoder1(), and 'TCM_decoder1()' (listed below) so that they can accommodate any (convolutional) encoder using the (octal) code generator instead of a function representing the state equation for the TCM encoder. Run the program "do_TCM_8PSK.m" with 'TCM()' replaced by 'TCM1()' and check if the routines work properly. Fill in the corresponding blanks of Table P9.9.

(d) With reference to the set partitioning shown in Fig. 9.14 (Sec. 9.4.5), why do you think the BER is affected by different constellations? Why is the TCM system of Fig. P9.9.1(b) less affected by different constellations than that of Fig. P9.9.1(a)?

```
function [outputs,state]=TCM_encoder1(Gc,input,state,Constellation)
% generates the output sequence of a binary TCM encoder
% Input:  Gc     = Code generator matrix consisting of octal numbers
%         K      = Number of input bits entering the encoder
%         Nsb    = Number of state bits of the TCM encoder
%         N      = Number of output bits of the TCM encoder
%         input  = Binary input message sequence
%         state  = State of the TCM encoder
%         Constellation = Signal sets for signal mapper
% Output: output = a sequence of signal points on Contellation
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
[K,N]=size(Gc);
for k=1:K, nsb(k)=length(de2bi(oct2dec(max(Gc(k,:))))-1; end
Nsb=sum(nsb);
tmp= rem(length(input),K);
input= [input zeros(1,(K-tmp)*(tmp>0))];
if nargin<3, state=zeros(1,Nsb);
 elseif length(state)==2^N
    Constellation=state; state=zeros(1,Nsb);
end
input_length= length(input);
N_msgsymbol= input_length/K;
input1= reshape(input,K,N_msgsymbol).';
outputs= [];
for l=1:N_msgsymbol
   ub= input1(l,:);
   is=1; nstate=[];
   output=zeros(1,N);
   for k=1:K
      tmp = [ub(k) state(is:is+nsb(k)-1)];
      nstate = [nstate tmp(1:nsb(k))]; is=is+nsb(k);
      for i=1:N
         output(i) = output(i) + ...
            deci2bin1(oct2dec(Gc(k,i)),nsb(k)+1)*tmp';
      end
   end
   state = nstate;
   output = Constellation(bin2deci(rem(output,2))+1);
   outputs = [outputs output];
end
```

```
function decoded_seq=TCM_decoder1(Gc,demod,Constellation,opmode)
% performs the Viterbi algorithm on the PSK demodulated signal
% Inpit: state_eq = External function for state equation saved
%                     in an M-file
%        K   = Number of input bits entering encoder at each cycle.
%        Nsb = Number of state bits of TCM encoder
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
[K,N]=size(Gc);
for k=1:K, nsb(k)=length(de2bi(oct2dec(max(Gc(k,:))))-1; end
Nsb=sum(nsb);
N_states=2^Nsb;
N_msgsymbol=length(demod);
for m=1:N_states
   for n=1:N_msgsymbol+1
      states(m,n)=0; % inactive in the trellis diagram
      p_state(m,n)=0; n_state(m,n)=0; input(m,n)=0;
   end
end
states(1,1)=1; % make the initial state active
cost(1,1)=0; K2=2^K;
for n=1:N_msgsymbol
  y=demod(n); %received sequence
  n1=n+1;
  for m=1:N_states
    if states(m,n)==1 %active
      xb=deci2bin1(m-1,Nsb);
      for m0=1:K2
        u=deci2bin1(m0-1,K);
        is=1; nstate=[]; output=zeros(1,N);
        for k=1:K
           tmp = [u(k) xb(is:is+nsb(k)-1)];
           nstate = [nstate tmp(1:nsb(k))]; is=is+nsb(k);
           for i=1:N
              output(i) = output(i) +
                 deci2bin1(oct2dec(Gc(k,i)),nsb(k)+1)*tmp';
           end
        end
        nxb(m0,:) = nstate;
        yb(m0) = Constellation(bin2deci(rem(output,2))+1);
        nxm0=bin2deci(nxb(m0,:))+1;
        states(nxm0,n1)=1;
        difference=y-yb(m0);
        d(m0)=cost(m,n)+difference*conj(difference);
        if p_state(nxm0,n1)==0
          cost(nxm0,n1)=d(m0);
          p_state(nxm0,n1)=m; input(nxm0,n1)=m0-1;
         else
          [cost(nxm0,n1),i]=min([d(m0) cost(nxm0,n1)]);
          if i==1, p_state(nxm0,n1)=m; input(nxm0,n1)=m0-1; end
        end
      end
    end
  end
end
```

```
decoded_seq=[];
if nargin<4 | ~strncmp(opmode,'term',4)
   % trace back from the best-metric state (default)
   [min_cost,m]=min(cost(:,n1));
  else m=1; %trace back from the all-0 state
end
for n=n1:-1:2
    decoded_seq= [deci2bin1(input(m,n),K) decoded_seq];
    m=p_state(m,n);
end
```

```
function [pemb,nombe,notmb]=
                TCM1(Gc,Constellation,SNRbdB,Target_no_of_error)
if nargin<4, Target_no_of_error=50; end
[K,N]=size(Gc); M=2^N; Rc=K/N;
Nf=288; % Number of bits per frame
Nmod=Nf/K; % Number of symbols per modulated frame
SNRb=10.^(SNRbdB/10); SNRbc=SNRb*Rc;
sqrtSNRc=sqrt(2*N*SNRbc); % Complex noise per K=Rc*N-bit symbol
nombe=0;   MaxIter=1e6;
for iter=1:MaxIter
   msg=randint(1,Nf); % Message vector
   coded = TCM_encoder1(Gc,msg,Constellation); % TCM encoding
   r= coded +(randn(1,Nmod)+j*randn(1,Nmod))/sqrtSNRc;
   decoded= TCM_decoder1(Gc,r,Constellation);
   nombe = nombe + sum(msg~=decoded(1:Nf));  %
   if nombe>Target_no_of_error, break; end
end
notmb=Nf*iter; % Number of total message bits
pemb=nombe/notmb; % Message bit error probability
```

9.10 DSTBC (Differential Space-Time Block Coding) with Four Transmit Antennas

The DSTBC with two transmit antennas (Sec. 9.4.8) can be extended into the DSTBC with four transmit antennas as follows:

The transmission matrix for sending $[x_0\ x_1\ x_2\ x_{3+0}\ x_{3+1}\ x_{3+2}\cdots x_{3n}\ x_{3n+1}\ x_{3n+2}\cdots]$ to the receiver is constructed as

$$S_{4(n+1)} = S_{4n} \bullet X_{3n} \quad \text{with} \quad S_0 = I_{4\times 4} \tag{P9.10.1}$$

where

$$X_{3n} = \begin{matrix}\text{Space}\downarrow\end{matrix}\begin{bmatrix}\overrightarrow{\text{Time}} & & & \\ x_{3n} & -x_{3n+1}^* & x_{3n+2}^*/\sqrt{2} & x_{3n+2}^*/\sqrt{2} \\ x_{3n+1} & x_{3n}^* & x_{3n+2}^*/\sqrt{2} & -x_{3n+2}^*/\sqrt{2} \\ x_{3n+2}/\sqrt{2} & x_{3n+2}/\sqrt{2} & -x_{3n,R}+jx_{3n+1,I} & -x_{3n+1,R}+jx_{3n,I} \\ x_{3n+2}/\sqrt{2} & -x_{3n+2}/\sqrt{2} & x_{3n+1,R}+jx_{3n,I} & -x_{3n,R}-jx_{3n+1,I} \end{bmatrix}\frac{1}{\sqrt{3}} \tag{P9.10.2}$$

Each row of this transmission matrix is transmitted through each antenna and each column is sent during each one of four consecutive periods. Then the signal received through a noise-free channel having frequency response $\mathbf{h}=[h_1\ h_2\ h_3\ h_4]$ can be described as

$$R_{4(n+1)}=\left[r_{4(n+1)}\ r_{4(n+1)+1}\ r_{4(n+1)+2}\ r_{4(n+1)+3}\right]=\mathbf{h}\,S_{4(n+1)} \overset{\text{(P9.10.1)}}{=} \mathbf{h}\,S_{4n}X_{4n}=R_{4n}X_{4n} \tag{P9.10.3}$$

The receiver uses the following decoding rule[W-10] to estimate the message sequence $\{x_n\}$:

$$\hat{\mathbf{x}}_{3n} = \begin{bmatrix} \hat{x}_{3n} \\ \hat{x}_{3n+1} \\ \hat{x}_{3n+2} \end{bmatrix} = \frac{\sqrt{3}}{|r_{4n}|^2+|r_{4n+1}|^2+|r_{4n+2}|^2+|r_{4n+3}|^2} \tag{P9.10.4}$$

$$\times \begin{bmatrix} r_{4n}^* r_{4(n+1)} + r_{4n+1} r_{4(n+1)+1}^* + \left((r_{4n+2}^* - r_{4n+3}^*)(-r_{4(n+1)+2} + r_{4(n+1)+3}) - (r_{4n+2} + r_{4n+3})(r_{4(n+1)+2}^* + r_{4(n+1)+3}^*) \right)/2 \\ r_{4n+1}^* r_{4(n+1)} - r_{4n} r_{4(n+1)+1}^* + \left((r_{4n+2}^* - r_{4n+3}^*)(r_{4(n+1)+2} + r_{4(n+1)+3}) + (r_{4n+2} + r_{4n+3})(-r_{4(n+1)+2}^* + r_{4(n+1)+3}^*) \right)/2 \\ \left(r_{4n+2}^*(r_{4(n+1)} + r_{4(n+1)+1}) + r_{4n+3}^*(r_{4(n+1)} - r_{4(n+1)+1}) + (r_{4n} + r_{4n+1}) r_{4(n+1)+2}^* + (r_{4n} - r_{4n+1}) r_{4(n+1)+3}^* \right)/\sqrt{2} \end{bmatrix}$$

Complete the following program "test_DSTBC_H4_PSK.m" and run it to see if this scheme works.

```
%test_DSTBC_H4_PSK.m
clear, clf
ITR=10; MaxIter=100;
sq2= sqrt(2); sq3= sqrt(3); sq6=sqrt(6);
M=4; phs= 2*pi*[0:M-1]/M; MPSKs= exp(j*phs); % 4-PSK symbols
h=[0.95*exp(j*0.5) exp(-j*0.4) 1.02*exp(-j*0.2) 0.98*exp(j*0.1)]; % Channel
dh= 0.01*[exp(-j*0.1) -exp(j*0.2) exp(j*0.1) -exp(-j*0.2)]; % Channel variation
Amp_noise = 0.2; % Amplitude of additive Gaussian noise
ner=0;
for itr1=1:ITR
   S= eye(4); % S0=I;
   r= h*S + Amp_noise*[randn(1,4)+j*randn(1,4)]; % Eq.(P9.10.3)
   x=[]; xh=[]; % Initialize the sequence of transmitted signal
   nn=[-3:0];
   for n=1:MaxIter
      nn= nn+4;
      xn= MPSKs(randint(1,3,M)+1); %  +-1+-j
      x= [x xn]; % Sequence of transmitted symbols
      x1=xn(1)/sq3; x2=xn(2)/sq3; x3=xn(3)/sq6; x3c=x3';
      x1R=real(x1); x1I=imag(x1); x2R=real(x2); x2I=imag(x2);
      X=[x1 -x2' x3c  x3c; x2  x1' x3c  -x3c;
       x3 x3 -x1R+j*x2I x2R+j*x1I; x3 -x3 -x2R+j*x1I -x1R-j*x2I]; %Eq.(P9.10.2)
      S = S*X; % Encoded signal Eq.(P9.10.1)
      r0=r; % Previously received signal
      rs = (h+dh*n)*S; % Received signal Eq.(P9.10.3)
      noise = Amp_noise*[randn(1,4)+j*randn(1,4)]; % Noise components
      r = rs + noise; % Received signal
      r034_=(r0(?)-r0(?))'; r034_r3=r034_*r(3); r034_r4=r034_*r(4);
      r034 = r0(?)+r0(?);   r034r3=r034*r(3)';  r034r4=r034*r(4)';
      xhn1= r0(?)'*r(?)+r0(2)*r(2)'+(-r034_r3+r034_r4-r034r3-r034r4)/2;
      xhn2= r0(2)'*r(?)-r0(?)*r(2)'+(r034_r3+r034_r4-r034r3+r034r4)/2;
      xhn3= (r0(3)'*(r(?)+r(2))+r0(4)'*(r(?)-r(2))...
              +(r0(?)+r0(2))*r(3)'+(r0(?)-r0(2))*r(4)')/sq2;
      xhns= [xhn1 xhn2 xhn3]*(sq3/(r0*r0')); % Decoded signal Eq.(P9.10.4)
      xhn= PSK_slicer(xhns,M);
      xh= [xh xhn];
   end
   ner= ner + sum(abs(x-xh)>0.1);
end
SER = ner/(MaxIter*3*ITR) % Symbol error rate
```

- Accordingly, we can use the complex convolution or modulation property (1.4.11) of CTFT to find the spectrum of a sinusoidal pulse $r_D(t)\cos(\omega_k t)$ of frequency $\omega_k = 2\pi k/T_B$ and duration $D=T_B$ as

$$r_D(t)\cos(\omega_k t) \overset{(11.2.1),(11.2.2)}{\underset{(1.4.11)}{\rightarrow}} \frac{1}{2\pi}\pi\{\delta(\omega-\omega_k)+\delta(\omega+\omega_k)\} * T_B \frac{\sin(\omega T_B/2)}{\omega T_B/2}$$
$$= \frac{T_B}{2}\left\{\frac{\sin((\omega-\omega_k)T_B/2)}{(\omega-\omega_k)T_B/2}+\frac{\sin((\omega+\omega_k)T_B/2)}{(\omega+\omega_k)T_B/2}\right\} = \frac{T_B}{2}\{\text{sinc}((f-f_k)T_B)+\text{sinc}((f+f_k)T_B)\},\ f_k = \frac{k}{T_B} \tag{11.2.3}$$

Fig. 11.3(b) based on these facts shows that the bandwidth of an N-subcarrier DFT-based OFDM signal is $(N+1)/T_B$ [Hz] where N is the DFT size. If each carrier is loaded with b bits of data, the data transmission bit rate is

$$\frac{Nb}{T_B+T_g}\text{[bits/s]}(T_B\text{: block interval, } T_g\text{: guard interval , i.e., cyclic prefix duration}) \tag{11.2.4}$$

so that the bandwidth efficiency is

$$\frac{Nb/(T_B+T_g)}{(N+1)/T_B} = \frac{N\ b}{N+1}\frac{T_B}{T_B+T_g}\text{ [bits/s/Hz]} \tag{11.2.5}$$

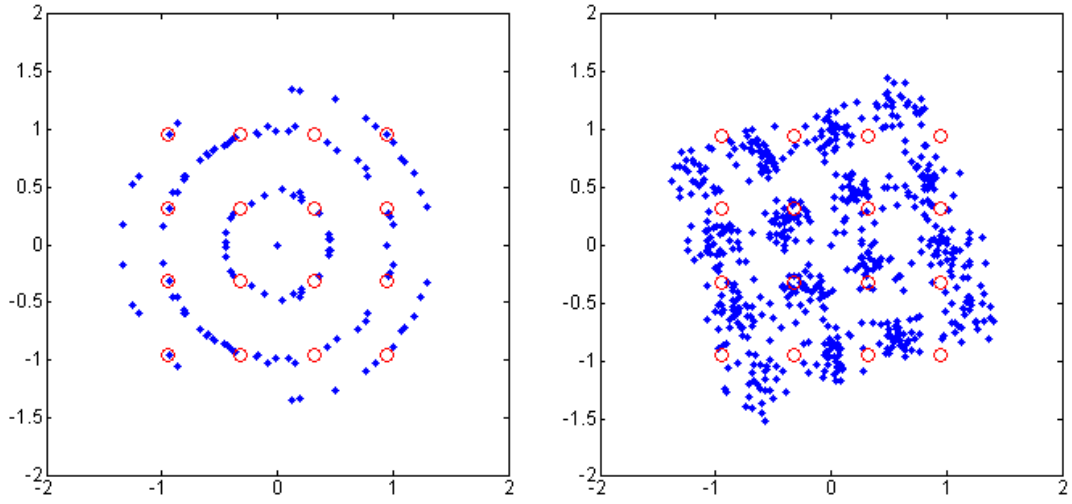
which becomes closer to the bandwidth efficiency b of the single-carrier communication as the number N of OFDM subcarriers increases and the guard interval T_g gets shorter.

11.3 CARRIER RECOVERY AND SYMBOL SYNCHRONIZATION

This topic was discussed in Chapter 8. To address the issue of carrier recovery and symbol timing on OFDM systems, let us begin with observing the effects of *STO* (*symbol time offset*) and *CFO* (*carrier frequency offset*). With this objective, we modify the MATLAB program "do_OFDM0.m" (in Section 11.1) by increasing the value of SNRbdB (in the 19th line) to 20 and activating the 11th line so that the STO amounting to one sample time is introduced. Running the modified program will yield a received signal constellation diagram like Fig. 11.4(a) and a much worse BER than that with neither STO nor CFO. Also, to take a look at the effect of CFO, activate the 14th line from the bottom (with the 11th line inactivated and SNRbdB=20) so that the CFO of 0.1 is introduced. This will yield a received signal constellation diagram like Fig. 11.4(b) and a worse BER. These effects of STO and CFO, called the *ISI* (*inter-symbol interference*) and *ICI* (*inter-carrier interference*), on the received signal can be observed and analyzed through the following expression of a time-domain received OFDM symbol:

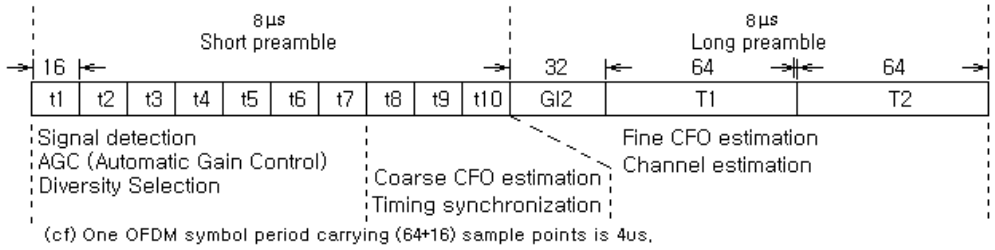
$$y^{(l)}[n] = \frac{1}{N}\sum_{k=0}^{N-1} G_k X_k^{(l)} e^{j2\pi(k+\varepsilon)(n+\delta)/N} + noise \tag{11.3.1}$$

where $y^{(l)}[n]$ denotes a (time-domain) received sequence for the l th OFDM symbol, G_k the channel frequency response (at the k th frequency index), $X_k^{(l)}$ the l th (frequency-domain) transmitted OFDM symbol, ε the CFO, δ the STO, and N the DFT size.

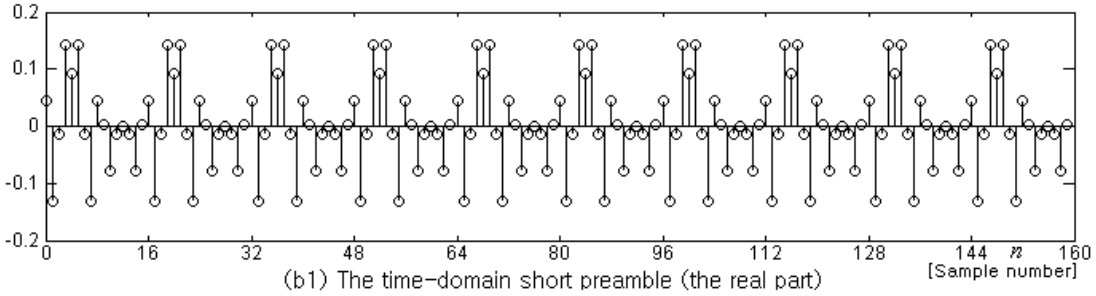


(a) With the STO of one sample (b) With the CFO of 0.1

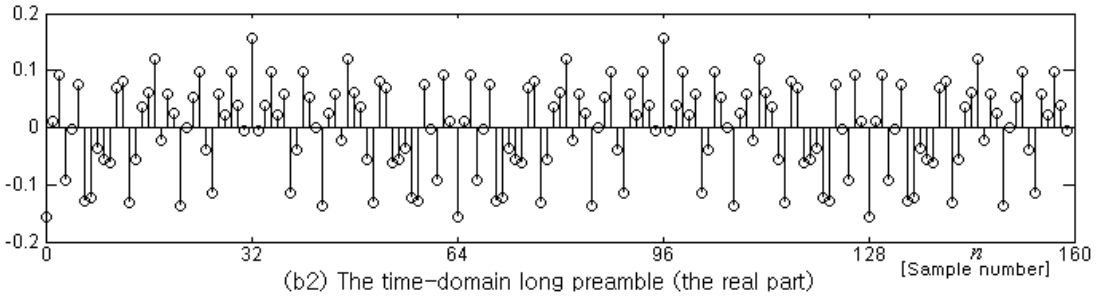
Figure 11.4 The signal constellation diagrams showing the effects of STO and CFO



(a) The structure of preamble in the IEEE Standard 802.11a



(b1) The time-domain short preamble (the real part)

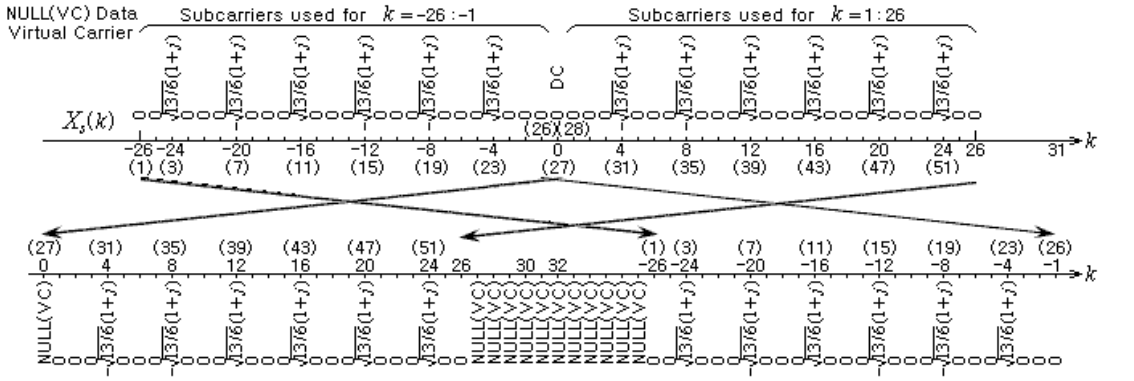


(b2) The time-domain long preamble (the real part)

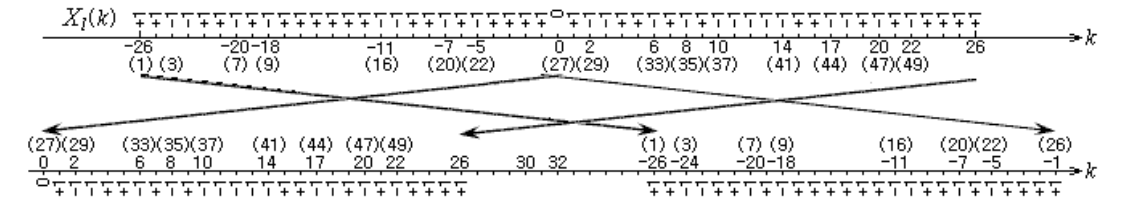
Figure 11.5 The time-domain preambles in the IEEE Standard 802.11a

```
function [short_preamble,Xs]=short_train_seq(Nfft)
if nargin<1, Nfft=64; end
sqrt136 = sqrt(13/6)*(1+i); Xs=zeros(1,53);
Xs([3 11 23 39 43 47 51]) = sqrt136; % Frequency domain for k=-26:26
Xs([7 15 19 31 35]) = -sqrt136;
xs = ifft([Xs(27:53) zeros(1,11) Xs(1:26)]); % Time domain
short_preamble = [xs xs xs(1:Nfft/2)];
```

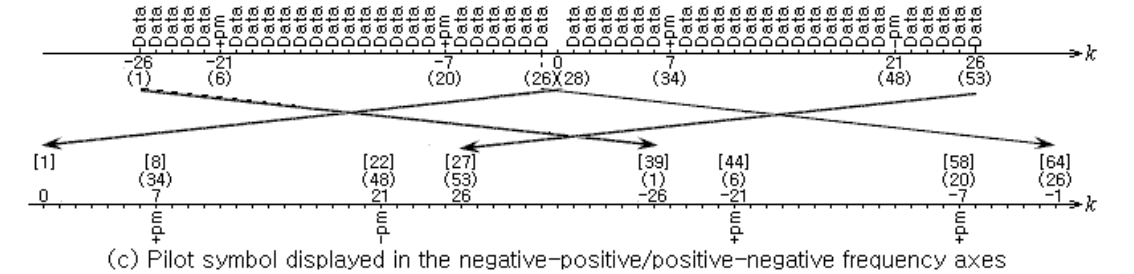
```
function [long_preamble,Xl]=long_train_seq(Nfft)
if nargin<1, Nfft=64; end
Xl([1:26 28:53]) = 1; % Frequency domain for k=-26:26
Xl([3 4 7 9 16 17 20 22 29 30 33 35 37:41 44 45 47 49])=-1;
xl = ifft([Xl(27:53) zeros(1,11) Xl(1:26)]); % Time domain
long_preamble= [xl(Nfft/2+1:Nfft) xl xl];
```



(a) Frequency-domain short preamble displayed in the negative-positive/positive-negative frequency axes



(b) Frequency-domain long preamble displayed in the negative-positive/positive-negative frequency axes



(c) Pilot symbol displayed in the negative-positive/positive-negative frequency axes

Figure 11.6 Short preamble, long preamble, and pilot symbol in the IEEE 802.11a standard

```
function phase_estimate=phase_from_pilot(outofFFT,pm)
% To estimate the phase offset based on the received pilot symbol
% outofFFT: OFDM symbol obtained from FFT at the RCVR
% pm      : the sign of pilot signals for each OFDM symbol
%           determined by the PN sequence
phase_estimate = angle(outofFFT([8 22 44 58])*[pm;-pm;pm;pm]);
```

```
function y=compensate_phase(x,phase)
% To compensate the received OFDM symbol x by phase deviation
y = x*exp(-j*phase);
```

Many carrier recovery schemes to overcome the carrier phase distortion and frequency offset and many symbol synchronization schemes to overcome the STO effect are implemented with the correlator, PLL, narrow-band filter, and/or tuner using the pilot symbol, phase reference symbol, training symbol (called a *preamble*), cyclic prefix (CP), and/or null symbol. For example, Fig. 11.5(a) shows the structure of the short and long OFDM training symbols (preambles) that is used in the IEEE Standard 802.11a[W-8, Sec. 17.3.3] where the short preamble consists of 10 repetitions of a (16-sample) 'short training sequence' and the long one consists of 2.5 repetitions of a (64-sample) 'long training sequence'. The above MATLAB routines 'short_train_seq()' and 'long_train_seq()' can be used to generate the short and long preambles in the time/frequency domain, respectively. Figs. 11.5(b1) and (b2) show the real parts of the short and long preambles that have been generated using each of the two routines, respectively. Figs. 11.6(a) and (b) show the frequency-domain short and long preambles, respectively, from which it can be seen that the energy of the frequency-domain long preamble $X_l(k)$ is 52 and the frequency-domain short preamble $X_s(k)$ has nonzero values of $\sqrt{13/6}(1+j)$ only for 12 subcarriers so that its energy is also $12\times(\sqrt{13/6}\,|1+j|)^2 = 52$. On the other hand, Fig. 11.6(c) shows that the pilot symbols (in the IEEE Standard 802.11a) are inserted in the $\{8^{\text{th}}, 22^{\text{th}}, 44^{\text{th}}, 58^{\text{th}}\}$ subcarriers (each corresponding to frequency indices $k=\{7,\ 21,-21,-7\}$, respectively,) of OFDM symbols. The above routines 'phase_from_pilot()' and 'compensate_phase()' estimate the phase offset based on the received pilot symbols and compensate the phase of the received signal by the phase estimate, respectively.

Not only the phase compensation but also the CFO (carrier frequency offset) compensation is needed for successful carrier recovery. In the IEEE Standard 802.11a, the fixed-lag correlation of the short training sequence is used for the coarse estimation of CFO and the fixed-lag correlation of the long training sequence is used for the fine estimation of CFO as follows:

$$\text{Coarse CFO estimate:}\quad \hat{\varepsilon}_c = \frac{N}{2\pi N_g}\angle\left\{\sum_{n=1}^{N_g} x_s[n+N_g]\,x_s^*[n]\right\} \tag{11.3.2a}$$

$$\text{Fine CFO estimate:}\quad \hat{\varepsilon}_f = \frac{1}{2\pi}\angle\left\{\sum_{n=1}^{N} x_l[n+N]\,x_l^*[n]\right\} \tag{11.3.2b}$$

where x_s is the (time-domain) short preamble, x_l the (time-domain) long preamble, N the FFT size and also the period of x_l, and N_g the length of the guard interval or cyclic prefix and also the period of x_s. Note that the estimable CFO ranges of the coarse and fine CFO estimations are

$$\frac{N}{2\pi N_g}[-\pi,+\pi] = \frac{64}{2\pi\times 16}[-\pi,+\pi] = [-2,+2] \quad\text{and}\quad \frac{1}{2\pi}[-\pi,+\pi] = [-0.5,+0.5] \tag{11.3.3}$$

respectively, where the latter estimate can be represented with less error by the same number of bits.

A question about the phase estimator (11.3.2) may arise in your mind. How can the information about the frequency offset be extracted from the time-domain correlation of the signal? To find the answer to this question, let us use Eq. (11.1.3) to write the values of, say, the short preamble $x_s[n]$ at two points apart from each other by $N_g=16$ samples as

$$x_s[n] \underset{\text{with CFO of } \varepsilon}{\overset{(11.1.3)}{=}} \frac{1}{N}\sum_{m=0}^{16-1} X_s(4m)e^{j2\pi(4m+\varepsilon)n/64}$$

$$\text{and } x_s[n+16] \underset{\text{with CFO of } \varepsilon}{\overset{(11.1.3)}{=}} \frac{1}{N}\sum_{m=0}^{16-1} X_s(4m)e^{j2\pi(4m+\varepsilon)(n+16)/64} \tag{11.3.4}$$

where we have taken into consideration that the spectrum $X_s(k)$ of the short preamble $x_s[n]$ has nonzero values only for $k=4m$ (see Fig. 11.6(a)). Multiplying one with the other's conjugate yields

$$x_s[n+16]x_s^*[n] = \frac{1}{N}\sum_{m=0}^{16-1} X_s(4m)\, e^{j2\pi(4m+\varepsilon)(n+16)/64}\, \frac{1}{N}\sum_{i=0}^{16-1} X_s^*(4i)e^{-j2\pi(4i+\varepsilon)n/64}$$

(The cross multiplication terms for $m \neq i$ disappears due to their orthogonality.)

$$= \frac{1}{N^2}\sum_{m=0}^{16-1} X_s(4m)X_s^*(4m)e^{j2\pi(4m+\varepsilon)(n+16)/64-j2\pi(4m+\varepsilon)n/64}$$

$$= \frac{16\times 13}{6N^2}(1+j)(1-j)e^{j2\pi(4m+\varepsilon)16/64} = \frac{13}{12\times 64}\angle(2m\pi+\frac{\pi}{2}\varepsilon) = K\angle\frac{\pi}{2}\varepsilon \tag{11.3.5}$$

which presents the basis of the coarse CFO estimator Eq. (11.3.2a). Similarly, the correlation of the long preamble yields

$$x_l[n+N]x_l^*[n] = \frac{1}{N}\sum_{m=0}^{N-1} X_l(m)e^{j2\pi(m+\varepsilon)(n+N)/N}\, \frac{1}{N}\sum_{i=0}^{N-1} X_s^*(i)e^{-j2\pi(i+\varepsilon)n/N}$$

(The cross multiplication terms for $m \neq i$ disappears due to their orthogonality.)

$$= \frac{1}{N^2}\sum_{m=0}^{N-1} X_l(m)X_l^*(m)e^{j2\pi(m+\varepsilon)(n+N)/N-j2\pi(m+\varepsilon)n/N}$$

$$= \frac{1}{N}e^{j2\pi(m+\varepsilon)} = \frac{1}{N}\angle(2m\pi+2\pi\varepsilon) = \frac{1}{N}\angle 2\pi\varepsilon \tag{11.3.6}$$

which presents the basis of the fine CFO estimator Eq. (11.3.2b). Note from Eqs. (11.3.2a) and (11.3.2b) that the phase is taken for the sum of the multiplication terms instead of the average. Why? Because the phase of a complex number does not depend on its magnitude. That is also why the average is not taken for the sum of the four pilot symbol terms in the routine 'phase_from_pilot()'.

The following routines 'coarse_CFO_estimate()' and 'fine_CFO_estimate()' can be used to get the coarse and fine CFO estimates via Eqs. (11.3.2a) and (11.3.2b), respectively. Note that the two routines differ in the frequency range to estimate, but not in the accuracy and therefore we can get a good CFO estimate using only the first one. However, to emphasize the quantitative difference between the two routines, it is assumed that an 8-bit number representation with relative resolution 2^{-8} is used for storing the CFO estimation result. The next MATLAB program "do_CFO.m" uses the routines 'short_train_seq()'/'long_train_seq()' to generate the short/long preambles, uses the routine 'set_CFO()' to set up a CFO, uses 'coarse_CFO_estimate()' and 'fine_CFO_estimate()' to estimate the CFO, and then uses 'compensate_CFO()' to compensate the CFO.

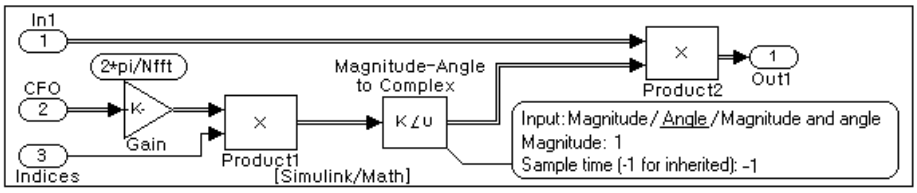
```
function coarse_CFO_est = coarse_CFO_estimate(tx,NB,Nw,STO)
% Input:
%    tx = Received signal
%    NB = Which block of size Nw to start computing correlation with?
%    Nw = Correlation window size
%    STO= Symbol Time Offset
% Output: coarse_CFO_est = Estimated carrier frequency offset
if nargin<4, STO = 0; end
if nargin<3, Nw = 16; end
if nargin<2, NB = 6; end
for i=1:2
   nn = STO + Nw*(NB+i) + [1:Nw];
   CFO_est(i) = angle(tx(nn+Nw)*tx(nn)'); % Eq.(11.3.2a)
end
CFO_est = sum(CFO_est)/pi; % Average
coarse_CFO_est = CFO_est - mod(CFO_est,4/128); % Stored with 8 bits
```

```
function fine_CFO_est = fine_CFO_estimate(tx,coarse_CFO_est,Nfft,STO)
% Input : tx = Received signal
%         coarse_CFO_est = coarse CFO estimate
%         Nfft = FFT size
%         STO  = Symbol Time Offset
% Output: fine_CFO_est = Estimated carrier frequency offset
if nargin<4, STO = 0; end
if nargin<3, Nfft = 64; end
if nargin<2, coarse_CFO_est = 0; end
tx1 = CFO_compensation(tx,coarse_CFO_est,Nfft,STO);
nn = STO + Nfft/2 + [1:Nfft];
cfo_est = angle(tx1(nn+Nfft)*tx1(nn)')/(2*pi); % Eq.(11.3.2b)
fine_CFO_est = CFO_est - mod(CFO_est,1/128); % Stored with 8 bits
```

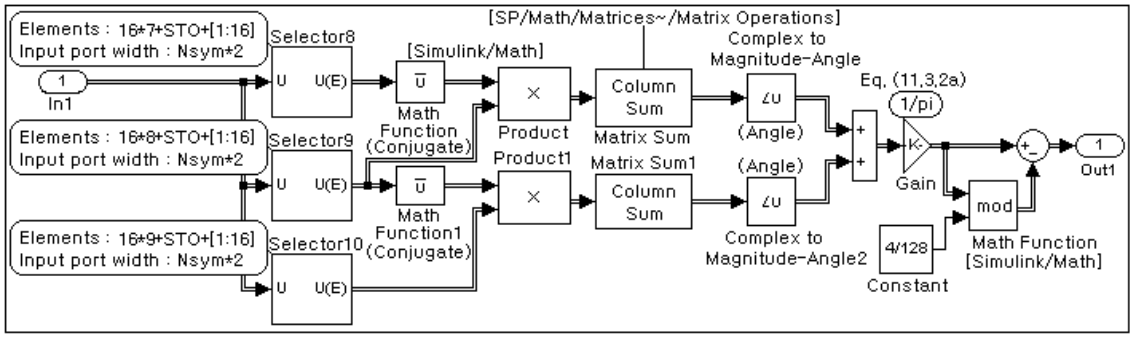
```
%do_CFO.m
clear, clf
Nfft = 64; Ng = 16;
CFO = 1.7; phase = 0; STO = -1; % CFO/Phase Offset/STO
NB = 6; % Which block to start computing the correlation with?
Nw = Ng; % Correlation window size
[short_preamble,S] = short_train_seq(Nfft);
[long_preamble,L] = long_train_seq(Nfft);
% Time-domain training symbol
tx = [short_preamble  long_preamble];
% Set up a pseudo CFO
tx_offset = set_CFO(tx,CFO,phase,Nfft);
% Coarse CFO estimation
coarse_CFO = coarse_CFO_estimate(tx_offset(1:160),NB,Nw,STO);
% Fine CFO estimation
fine_CFO = fine_CFO_estimate(tx_offset(161:320),coarse_CFO,Nfft);
% Overall CFO estimate
CFO_estimate = coarse_CFO + fine_CFO;
form1 = '\n For CFO=%10.8f, CFO estimate(%10.8f) = ';
form2 = 'coarse estimate(%10.8f)+fine estimate(%10.8f)\n';
fprintf([form1 form2],CFO,CFO_estimate,coarse_CFO,fine_CFO);
% CFO compensated symbols
tx_compensated = compensate_CFO(tx_offset,CFO_estimate,Nfft,STO);
discrepancy = norm(tx-tx_compensated)/length(tx)
```

```
function tx_with_CFO = set_CFO(tx,CFO,phase,Nfft,STO)
% CFO/phase = pseudo Carrier frequency/phase offset (pretended)
% STO       = Symbol Time Offset
if nargin<5, STO = 0; end % Symbol Time Offset
if nargin<4, Nfft = 64; end
if nargin<3, phase = 0; end
n = -STO + [0:length(tx)-1];
tx_with_CFO = tx.*exp(j*(2*pi*CFO/Nfft*n+phase));
```

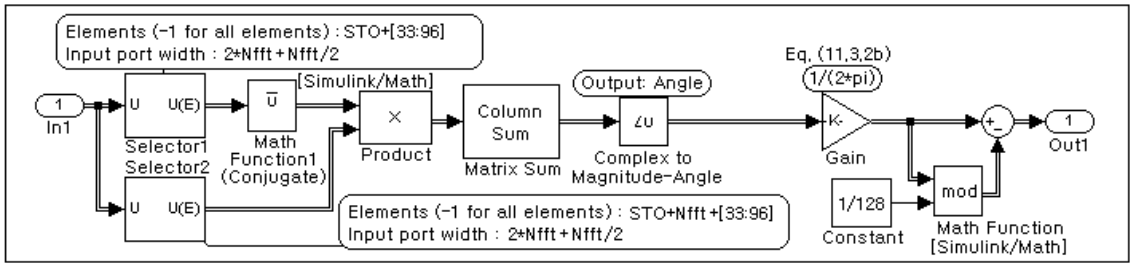
```
function CFO_compensated = compensate_CFO(tx,CFO_est,Nfft,STO)
% CFO_est = Estimated carrier frequency offset
% STO     = Symbol Time Offset
if nargin<4, STO = 0; end
if nargin<3, Nfft = 64; end
n = -STO + [0:length(tx)-1]; %m = -STO+m0 +[0:length(tx)-1];
CFO_compensated = tx.*exp(-j*2*pi*CFO_est/Nfft*n);
```



(a) A subsystem for setting or compensating CFO (carrier frequency offset)



(b) A subsystem for coarse CFO estimation based on the short preamble



(c) A subsystem for fine CFO estimation based on the long preamble

Figure 11.7 Subsystems to be used for CFO estimation and compensation

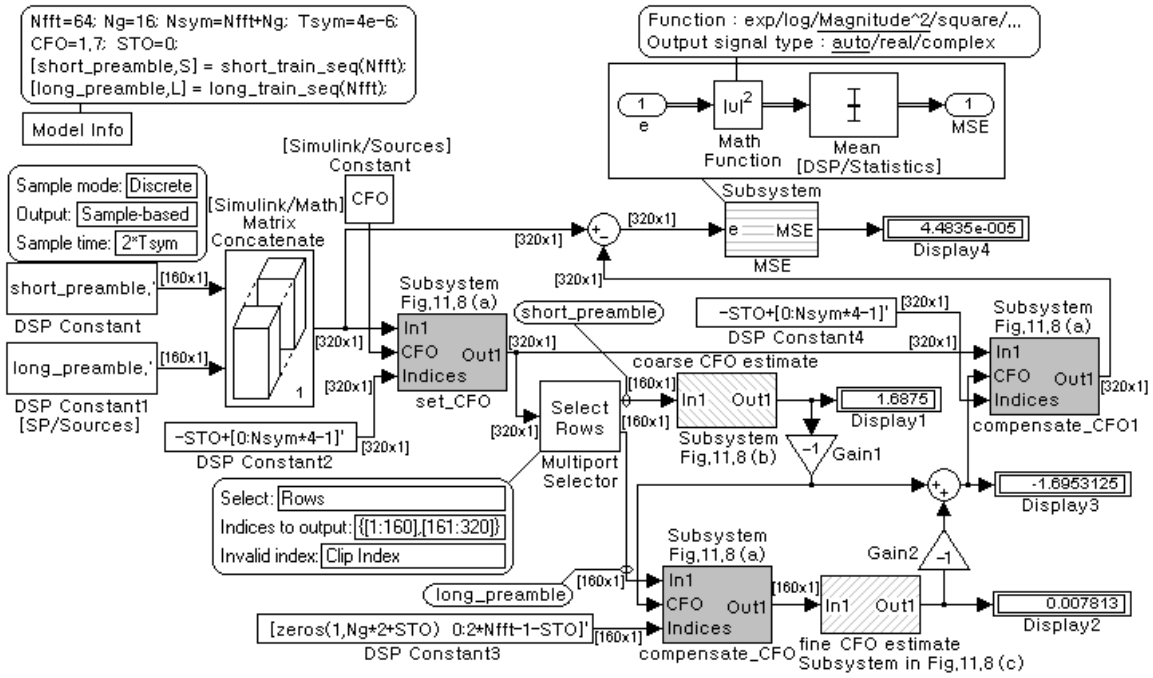


Figure 11.8 Simulink model for demonstrating CFO estimation and compensation ("CFO_sim.mdl")

Fig. 11.7 shows the three Simulink subsystems that can be used to estimate and compensate the CFO in a Simulink model for OFDM system simulation. Note the following about the subsystems:

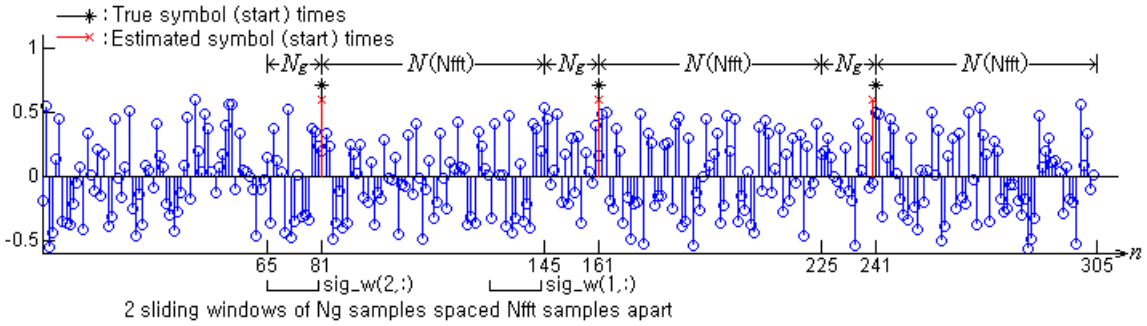
- The subsystem of Fig. 11.7(a) is equivalent to the MATLAB routine 'set_CFO()', but it can be made to work like 'compensate_CFO()' by negating the CFO input as can be seen in Fig. 11.8.
- The subsystem of Fig. 11.7(b) is equivalent to the MATLAB routine 'coarse_CFO_estimate()', which uses Eq. (11.3.2a) to make a coarse CFO estimation based on the two correlation values, one between the 8th (16-sample) block and the 9th block and the other between the 9th block and the 10th block of the received sequence corresponding to the (160-sample) short preamble.
- The subsystem of Fig. 11.7(c) is equivalent to 'fine_CFO_estimate()', which uses Eq. (11.3.2b) to make a fine CFO estimation based on the correlation value between the 1st (64-sample) block and the 2nd block of the received sequence corresponding to the (160-sample) long preamble.

Fig. 11.8 shows a Simulink model named "CFO_sim.mdl", which demonstrates the CFO estimation and compensation using the short and long preambles and the three subsystems depicted in Fig. 11.7. About this Simulink model, note the following:

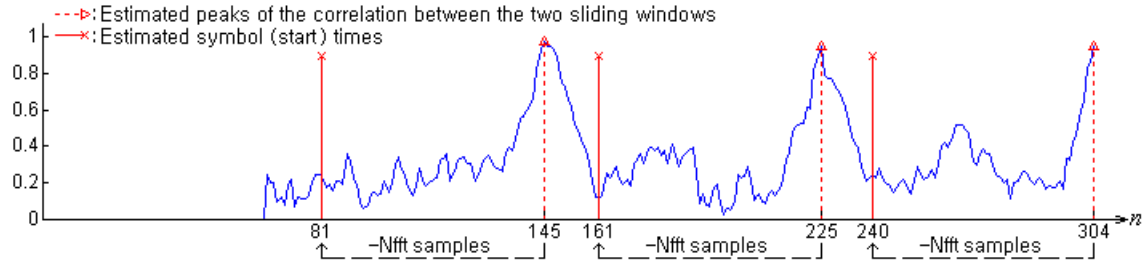
- The coarse CFO estimate is computed by the (subsystem) block 'coarse CFO estimate' and then is used to compensate the CFO of the long preamble through the block 'compensate_CFO'.
- The fine CFO estimate (obtained from the correlation value of the coarse-CFO-compensated long preamble by the block 'fine CFO estimate') and the coarse CFO estimate are negated, fed into the block 'compensate_CFO1', and then used to compensate the CFO of a general signal.

Interested readers are invited to compose the Simulink model "CFO_sim.mdl" and run it to see if the CFO is well tracked for different values of CFO and consequently, the mean square error between the original signal and the CFO-inserted-and-compensated one (computed by the block 'MSE') is very small.

```
%do_STO_estimation.m
% To estimate the STO (Symbol Time Offset)
%Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
clear, clf
Cor_thd = 0.93; % Threshold of correlation for peak detection
Nfft=64;  Ng=16; % FFT/Guard interval size
Nsym=Nfft+Ng;  Nw=Ng; % Symbol/Window size
Nd=65; % Remaining period of the last symbol in the previous frame
% Make a pseudo frame to transmit
t_frame = rand(1,Nd)-0.5+j*(rand(1,Nd)-0.5);
N_Symbols = 3; % Number of OFDM symbols to be generated for simulation
for i=1:N_Symbols
   symbol = rand(1,Nfft)-0.5+j*(rand(1,Nfft)-0.5); % An arbitrary data
   symbol_cp = [symbol(end-Ng+1:end) symbol]; % OFDM Symbol with CP
   t_frame = [t_frame symbol_cp]; % Append a frame by a symbol with CP
end
L_frame = length(t_frame);  nn=0:L_frame-1;
noise = 0.2*(rand(1,L_frame)-0.5 +j*(rand(1,L_frame)-0.5));
r = t_frame + noise;
sig_w = zeros(2,Nw); % Initialize the two sliding window buffers
STOs = [0]; % Initialize the STO buffer
for n=1:L_frame
   sig_w(1,:) = [sig_w(1,2:end) r(n)]; % Update signal window 1
   m = n-Nfft;
   if m>0
     sig_w(2,:)=[sig_w(2,2:end) r(m)]; % Update signal window 2
     den = norm(sig_w(1,:))*norm(sig_w(2,:));
     corr(n) = abs(sig_w(1,:)*sig_w(2,:)')/den;
     if corr(n)>Cor_thd & m>STOs(end)+Nsym-15
       STOs=[STOs  m]; % List the estimated STO
     end
   end
end
Estimated_STOs = STOs(2:end)
True_STOs = Nd+Ng + [0:N_Symbols-1]*Nsym
subplot(311)
stem(nn,real(r)), ylim([-0.6  1.1])
hold on, stem(True_STOs,0.8*ones(size(True_STOs)),'k*')
stem(Estimated_STOs,0.6*ones(size(Estimated_STOs)),'rx')
title('Estimated Starting Times of OFDM Symbols')
subplot(312)
plot(nn+1,corr), ylim([0  1.2]), hold on,
stem(Estimated_STOs+Nfft,corr(Estimated_STOs+Nfft),'r:^')
% The points at which the correlation is presumably maximized,
%  yielding the STO estimates.
stem(Estimated_STOs,0.9*ones(size(Estimated_STOs)),'rx')
title('Correlation between two sliding windows across Nfft samples')
set(gca,'XTick',sort([Estimated_STOs Estimated_STOs+Nfft]))
```



(a) True starting times and estimated ones of OFDM symbols each of which consists of (N_g+N) samples



(b) Correlation between two sliding windows (of size Ng) apart from each other by Nfft samples

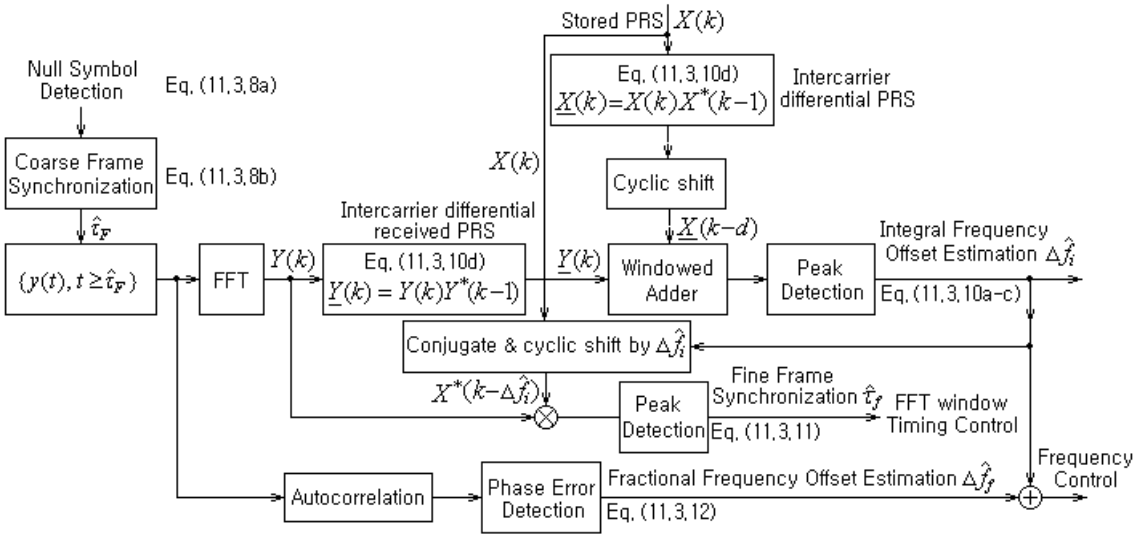
Figure 11.9 STO (symbol time offset) estimation

The above program "do_STO_estimation.m" demonstrates how the correlation value can be used to estimate the STO of an OFDM symbol consisting of N_g (guard interval size) cyclic prefix (CP) samples and N (FFT size) signal samples. As shown in Fig. 11.9(a), it keeps updating a $2\times N_g$ matrix 'sig_w' that contains the N_g samples of two sliding windows in each of its two rows, finds the (local) peak times of the crosscorrelation value between two row vectors $\mathbf{x}_{1,n}=x(n-N_g+1:n)$ and $\mathbf{x}_{2,n}=x(n-N-N_g+1:n-N)$ (corresponding to two sliding windows), and sets the times (`m=n-Nfft`) of N samples before the local peak times to the STO estimates (to be stored in the vectors named 'STOs' and 'Estimated_STOs'). In Fig. 11.9(b), the measured peak times of the correlation value and the corresponding STO estimates are denoted by dotted lines and solid lines, respectively. Note that the on-line detection of local maxima or minima of a noisy signal is not so simple since there may be spurious peaks around true peaks due to the noise. That is why the correlation value is divided by the product of the norms of the two vectors for normalization as

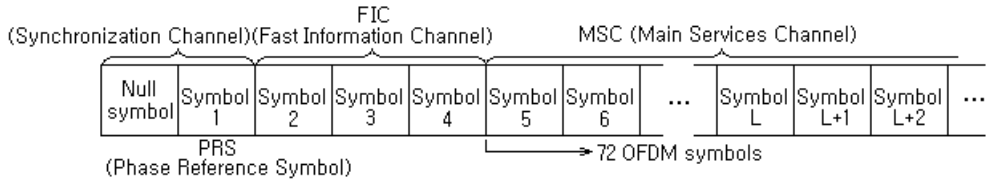
$$\phi_y[n]=\frac{\left|\sum_{m=n-N_g+1}^{n} x[m]x^*[m-N]\right|}{\sqrt{\sum_{m=n-N_g+1}^{n}|x[m]|^2}\sqrt{\sum_{m=n-N_g+1}^{n}|x[m-N]|^2}}=\frac{|\text{sig\_w}(1,:)\,\text{sig\_w}(2,:)'|}{||\text{sig\_w}(1,:)||||\text{sig\_w}(2,:)||} \tag{11.3.7}$$

('(apostrophe): the MATLAB operator representing the conjugate transpose, i.e., Hermitian)

and a threshold of, say, Cor_thd=0.93 is used to determine if the presumable peak time has been reached or not. The additional condition (m>STOs(end)+ $N_{sym}-15$) to determine peak times has been required not to let another (most probably wrong) peak reported within $N_{sym}-15$ ($N_{sym}=N_g+N$: *OFDM symbol duration*) samples after a peak is reported.



(a) Block diagram of the synchronization scheme for the EUREKA-147 DAB system



(b) The EUREKA-147 transmission frame structure of 96ms for transmission mode I

Figure 11.10 The synchronization scheme and transmission frame of the EUREKA-147 DAB system

Table 11.1 Transmission modes of the EUREKA-147 DAB system

Transmission mode	I (Large area)	II (Topographical situation)	III (Satellite transmission)	IV (Limited speed and direct line of sight)
Maximum carrier frequency	375 MHz	1.5 GHz	3 GHz	1.5 GHz
Number of subcarriers	1,536	384	192	768
Subcarrier interval frequency	1 kHz	4 kHz	8 kHz	2 kHz
Guard interval time	246 us	62 us	31 us	123 us
OFDM symbol period	1 ms	250 us	125 us	500 us
Frame period	96 ms	24 ms	24 ms	48 ms

Now, let us consider the synchronization scheme that is used in the EUREKA-147 standard for DAB (digital audio broadcasting). It consists of the four processes working in coordination, i.e. the coarse frame synchronization, the IFO (integral frequency offset) estimation, the fine frame synchronization, and the FFO (fractional frequency offset) estimation, as depicted by the block diagram of Fig. 11.10(a). Fig. 11.10(b) shows the EUREKA-147 transmission frame structure of 96ms for transmission mode I where the null symbol and *PRS* (*phase reference symbol*) symbol preceding the frame can be used for the frame synchronization.

1. Coarse Frame Synchronization

The tactics to estimate the starting points of a null symbol and a frame based on the energy ratio between the two adjacent windows of size N_w can be expressed by the following equations:

$$\text{Null time estimate: } \hat{n}_N = \text{Arg}\underset{n-N_w}{\text{Min}} \frac{\sum_{m=n-N_w+1}^{n} r_m^* r_m}{\sum_{m=n-N_w+1}^{n} r_{m-N_w}^* r_{m-N_w}} \tag{11.3.8a}$$

$$\text{Frame time estimate: } \hat{n}_F = \text{Arg}\underset{n-N_w}{\text{Max}} \frac{\sum_{m=n-N_w+1}^{n} r_m^* r_m}{\sum_{m=n-N_w+1}^{n} r_{m-N_w}^* r_{m-N_w}} \tag{11.3.8b}$$

The estimated null symbol period $(\hat{n}_F - \hat{n}_N)$ can be used to detect the transmission mode since it depends on the transmission mode (see Table 11.1).

The following program "do_sync_w_double_window.m" simulates the process of catching the starting points of a null symbol and the frame (preceded by the null symbol) based on the energy ratio between two successive blocks of size N_{null}. It also simulates the process of estimating the starting points of each OFDM symbol based on the correlation value between two blocks of size N_g spaced N_{fft} samples apart (N_{fft} : FFT size). To minimize the number of computations as well as to save the memory, we maintain several windows (buffers) to store some duration of signal samples, powers, energies, and correlations as follows: (see Fig. 11.11)

(1) $\text{power\_w}[n] = r[n]^* r[n]$ with size of $N_{null}+1$

(2) $\text{energy\_w1}[n] = \sum_{m=n-N_{null}+1}^{n} \text{power\_w}[m] = \text{energy\_w1}[n-1] + \text{power\_w}[n] - \text{power\_w}[n-N_{null}]$
$= \text{energy\_w1}[n-1] + \text{power\_w}[\text{end}] - \text{power\_w}[1]$ with size of $N_{null}+1$

(3) $\text{sig\_w}[n] = r[n]$ with size of $N_{fft}+1$

(4) $\text{corr\_w}[n] = r[n]^* r[n-N_{fft}] = \text{sig\_w}[\text{end}]^* \text{sig\_w}[1]$ with size of N_g+1

(5) $\text{energy\_w2}[n] = \sum_{m=n-N_g+1}^{n} \text{power\_w}[m] = \text{energy\_w2}[n-1] + \text{power\_w}[n] - \text{power\_w}[n-N_g]$
with size of $N_{fft}+1$

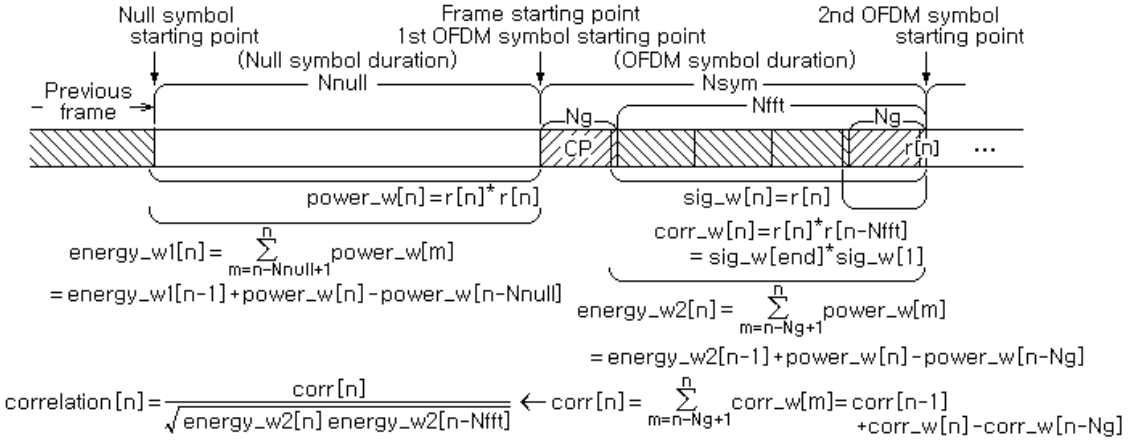


Figure 11.11 Window buffers maintained to catch the starting points of the null, frame, and OFDM symbols

```
%do_sync_w_double_window.m
% Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
clear, clf
Cor_thd=0.988; % The threshold to determine the peak of correlation
Nfft=64; Ng=16; Nsym=Nfft+Ng; Nsym1=Nsym+1;
Nnull=Nsym; Nw=Nnull; Nw1=Nw+1; Nw2=Nw*2; Ng1=Ng+1; Nfft1=Nfft+1;
Nd=90; % Remaining period of the last symbol in the previous frame
N_OFDM=3; % One Null + N_OFDM symbols
Max_energy_ratio=0;  Min_energy_ratio=1e10;
r = [rand(1,Nd)-0.5+j*(rand(1,Nd)-0.5) zeros(1,Nnull)];
for i=1:N_OFDM
   symbol=rand(1,Nfft)-0.5 +j*(rand(1,Nfft)-0.5);
   r = [r symbol(end-Ng+1:end) symbol];
end
L_frame = length(r);
r = r + 0.1*(rand(1,L_frame)-0.5+j*(rand(1,L_frame)-0.5));
energy_w1=zeros(1,Nw1); power_w=zeros(1,Nw1);
sig_w=zeros(1,Nfft1); energy_w2=zeros(1,Nfft1); corr_w=zeros(1,Ng1);
OFDM_start_points = [0]; corr=0;
for n=1:L_frame
   sig_w = [sig_w(2:end) r(n)]; % Signal window
   power_n = r(n)'*r(n); % Current signal power
   power_w = [power_w(2:end)  power_n]; % Power window
   energy_w1=[energy_w1(2:end) energy_w1(end)+power_n]; %Energy window
   if n>Nw, energy_w1(end)=energy_w1(end)-power_w(1); end %of size Nw
   energy_w2=[energy_w2(2:end) energy_w2(end)+power_n]; %Energy window
   if n>Ng, energy_w2(end)=energy_w2(end)-power_w(end-Ng); end
   corr_w(1:end-1) = corr_w(2:end);
   if n>Nfft
      %Correlation between signals at 2 points spaced Nfft samples apart
      corr_w(end)=abs(sig_w(end)'*sig_w(1)); corr=corr+corr_w(end);
   end
   if n>Nsym, corr=corr-corr_w(1); end %Correlation window of Ng pts
   % Null Symbol detection based on energy ratio
   if n>=Nw2
      energy_ratio = energy_w1(end)/energy_w1(1);
      energy_ratios(n) = energy_ratio;
      if energy_ratio<Min_energy_ratio  % Eq.(11.3.8a)
        Min_energy_ratio = energy_ratio; Null_start_point = n-Nw+1;
      end
      if energy_ratio>Max_energy_ratio  % Eq.(11.3.8b)
        Max_energy_ratio = energy_ratio; F_start = n-Nw+1;
      end
   end
   % CP-based Symbol Time estimation
   if n>Nsym
      % Normalized, windowed correlation across Nfft samples for Ng pts
      correlation=corr/sqrt(energy_w2(end)*energy_w2(1)); % Eq.(11.3.9)
      correlations(n) = correlation;
      if correlation>Cor_thd&n-Nsym>OFDM_start_points(end)+Nfft
        OFDM_start_points = [OFDM_start_points n-Nsym+1];
      end
    end
end
```

```
Estimated_start_points=
    [Null_start_point F_start OFDM_start_points(2:end)]
True_start_points=[Nd+1:Nsym:L_frame]
N_True_start_points=length(True_start_points);
subplot(311), stem(real(r)), set(gca,'XTick',True_start_points)
hold on, stem(True_start_points,0.9*ones(1,N_True_start_points),'k*')
N_Sym=length(Estimated_start_points);
stem(Estimated_start_points,1.1*ones(1,N_Sym),'rx')
title('Estimated Starting Points of Symbols')
subplot(312),semilogy(energy_ratios),set(gca,'XTick',True_start_points)
title('Ratio of 2 Successive Windowed Energies for Nw samples')
subplot(313), plot(correlations)
hold on, title('Correlation across Nfft samples')
```

At every instant when a sampled signal arrives, the normalized and windowed correlation value

$$\text{correlation}[n] = \frac{\sum_{m=n-N_g+1}^{n} \text{corr\_w}[m]}{\sqrt{\text{energy\_w2}[n]\,\text{energy\_w2}[n-N_{fft}]}} \overset{?}{>} \text{Threshold} \tag{11.3.9}$$

is computed to determine if it is the correlation value between the CP and the last N_g samples of an OFDM symbol, i.e. if the current sample is the end of an OFDM symbol or not. If the normalized correlation value is found to exceed some threshold, say, 0.988 at n, the starting point of the detected OFDM symbol is determined to be $n-N_{sym}+1$ (one OFDM symbol duration before the detection time) where $N_{sym}=N_g+N_{fft}$ is the *OFDM symbol duration*. Compared with this symbol timing process, catching the starting points of a null symbol and a frame based on the energy ratio between two successive blocks is very misty since there can be no normalization of energy ratio and accordingly, the appropriate threshold values to determine the maximum and minimum of the energy ratio are difficult to fix. Let us run the MATLAB program "do_sync_w_double_window.m" to get Fig. 11.12 together with the following result:

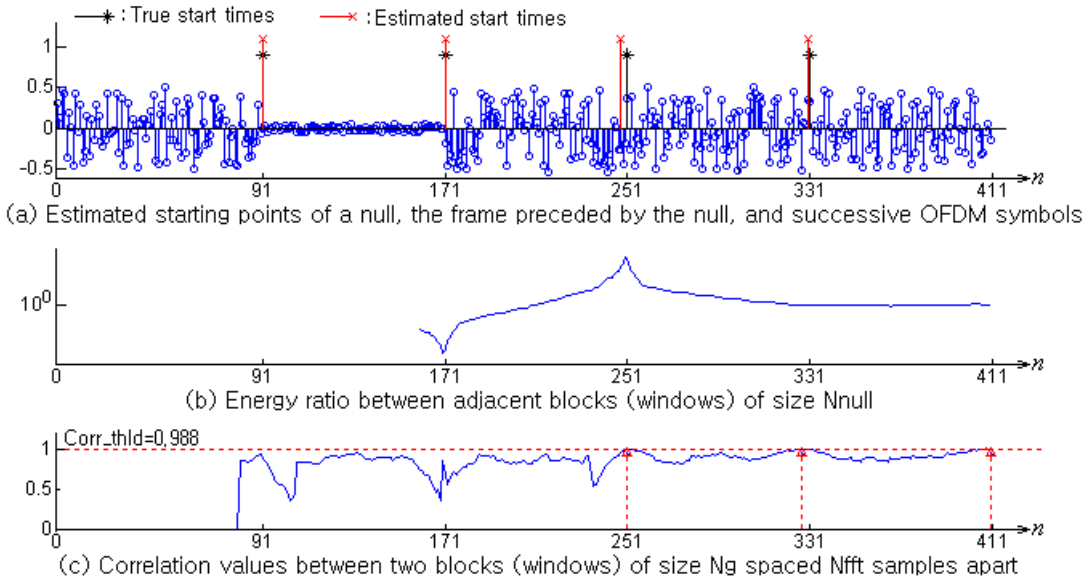


Figure 11.12 An example of simulation results obtained by running "do_sync_w_double_window.m"

```
>> do_sync_w_double_window
   Estimated_start_points  =    91   171   171   248   330
   True_start_points  =    91   171   251   331
```

The detection of the starting points of a null, the frame preceded by the null symbol, and successive OFDM symbols seems to be successful. That is right as far as the OFDM symbol starting points are concerned. However, the maximum/minimum points of the energy ratio for estimating the starting points of a null/frame (shown in Fig. 11.12(b)) have not been detected on-line since they could be recognized only after a considerable time span. In this aspect, the above program has a room for improvement towards more practical detection of the starting points of a null and the frame preceded by the null symbol, possibly in concert with the OFDM symbol starting point detection.

2. Integral Frequency Offset Estimation

The *CFO* (*carrier frequency offset*) that may be caused by the mismatch of the oscillators in the transmitter and receiver can be regarded as the sum of the *IFO* (*integral frequency offset*) and the *FFO* (*fractional frequency offset*) where the IFO and the FFO are an integer and a fraction times the subcarrier frequency interval $\Omega_0 = 2\pi/N_{fft}$, respectively. The IFO can be estimated based on the correlation between the received PRS(phase reference symbol)-FFT signal $Y_{PRS}(k)$ and the local (frequency-domain) PRS signal $X_{PRS}(k)$ at the receiver by applying one of the following three methods:

APRS (Algorithm using PRS):

$$\Delta\hat{f}_i = \underset{d}{\text{ArgMax}} \left| \sum_{k=0}^{N_{fft}-1} Y_{PRS}(k) X_{PRS}^*(k-d) \right| \tag{11.3.10a}$$

ACIR (Algorithm using the channel impulse response):

$$\Delta\hat{f}_i = \underset{d}{\text{ArgMax}} \left\{ \underset{n}{\text{Max}}\, z[n] = \text{IFFT}\left\{ Y_{PRS}(k) X_{PRS}^*(k-d) \right\} \right\} \tag{11.3.10b}$$

AIDC (Algorithm using the intercarrier differential correlation):

$$\Delta\hat{f}_i = \underset{d}{\text{ArgMax}} \left| \sum_{k=0}^{N_{fft}-1} \underline{Y}_{PRS}(k) \underline{X}_{PRS}^*(k-d) \right| \tag{11.3.10c}$$

where

$$\underline{Y}(k) = Y(k)Y^*(k-1), \quad \underline{X}(k) = X(k)X^*(k-1) \tag{11.3.10d}$$

3. Fine Frame Synchronization

The *fine frame synchronization* can be performed by taking the peak point of the correlation between the received signal $y_{PRS}[n]$ and the time-domain PRS $x_{PRS}[n]$ where the (time-domian) correlation can equivalently be computed from the IFFT of the (frequency-domain) product of the received FFT signal $Y(k)$ and the IFO-compensated PRS $X_{PRS}^*(k-\Delta\hat{f}_i)$:

$$\hat{n}_f = \text{Arg}\,\underset{n}{\text{Max}} \left\{ z[n] = \text{IFFT}\left\{ Y(k) X^*(k-\Delta\hat{f}_i) \right\} \right\} \tag{11.3.11}$$

4. Fractional Frequency Offset Estimation

With the same idea as Eq. (11.3.2b), the fractional frequency offset (FFO) estimate can be obtained from $1/2\pi$ times the phase difference between the two (fine frame-synchronized) blocks of size N_w spaced N_{fft} samples apart:

$$\Delta\hat{f}_f = \frac{1}{2\pi}\angle\left\{\sum_{n=1}^{N_w} y[n+\hat{\tau}_f+N_{fft}]\,y^*[n+\hat{\tau}_f]\right\} \tag{11.3.12}$$

where the two blocks are supposed to agree with each other without CFO and noise and the window size N_w is often set equal to the guard interval or cyclic prefix (CP) size N_g. Note that a large window size will help increasing the accuracy of the FFO estimation.

```
function IFO_est=IFO_estimate(y,X,IFO_range)
% To estimate the IFO (Integral Frequency Offset)
% y: A received time-domain signal, supposedly containing ifft(PRS)
% X: (frequency-domain) Phase Reference Symbol
% IFO_range : Range of possible IFOs to be searched
M=3; Max=zeros(1,M); IFO_est=zeros(1,M); % 3 methods to estimate IFO
Nfft=length(X);  Y = fft(y,Nfft);
Ybar=Y.*conj(Y([Nfft 1:Nfft-1])); % Eq.(11.3.10)
Xbar=X.*conj(X([Nfft 1:Nfft-1]));
for i=1:length(IFO_range)
   d=IFO_range(i);  YX = Y.*conj(rotate_r(X,d));
   Mag(1) = abs(sum(YX)); % Eq.(11.3.10a) APRS
   Mag(2) = max(abs(ifft(YX))); % Eq.(11.3.10b) ACIR
   Mag(3) = abs(Ybar*rotate_r(Xbar,d)'); % Eq.(11.3.10c) AIDC
   for m=1:M
      if Mag(m)>=Max(m),  Max(m)=Mag(m); IFO_est(m)=d;  end
   end
end
```

```
function PRS=phase_ref_symbol()
% Nfft=Nsd+Nvc=1536+512=2048;
Nfft=1536+512; %Nsd(# of data subcarriers)+Nvc(# of virtial carrier)
h= ...
[0 2 0 0  0 0 1 1  2 0 0 0  2 2 1 1  0 2 0 0  0 0 1 1  2 0 0 0  2 2 1 1;
 0 3 2 3  0 1 3 0  2 1 2 3  2 3 3 0  0 3 2 3  0 1 3 0  2 1 2 3  2 3 3 0;
 0 0 0 2  0 2 1 3  2 2 0 2  2 0 1 3  0 0 0 2  0 2 1 3  2 2 0 2  2 0 1 3;
 0 1 2 1  0 3 3 2  2 3 2 1  2 1 3 2  0 1 2 1  0 3 3 2  2 3 2 1  2 1 3 2;
0 2 0 0  0 0 1 1  2 0 0 0  2 2 1 1  0 2 0 0  0 0 1 1  2 0 0 0  2 2 1 1];
n=[1 2 0 1  3 2 2 3  2 1 2 3  1 2 3 3  2 2 2 1  1 3 1 2 ...
   3 1 1 1  2 2 1 0  2 2 3 3  0 2 1 3  3 3 3 0  3 0 1 1];
jpi2 = j*pi/2;
for p=1:12 %12*4*32=1536
  for i=1:4
    if p<=6, i1=i; else i1=6-i; end
    for k=1:32
       temp_seq((p-1)*128+(i-1)*32+k)=exp(jpi2*(h(i1,k)+n((p-1)*4+i)));
    end
  end
end
PRS = [zeros(1,256) temp_seq(1:768) 0 temp_seq(769:end) zeros(1,255)];
```

```
%do_sync_for_DMB.m
clear, clf
Nfft=2048; Ng=504; Nnull=2656;
CFO = -1.7;  phase = 0; % A pseudo Carrier Frequency/Phase Offset
nF = 1 % A pseudo Frame Time Offset (delay)
PRS = phase_ref_symbol;
prs = ifft(PRS,Nfft); % Time-domain PRS
tx = [prs(Nfft-Ng+1:Nfft) prs]; % Add Cyclic Prefix
L_tx=length(tx);
% Set up the (pseudo) CFO (pretended)
Nd = 100; % Delay tolerance
y = set_CFO(tx,CFO,phase,Nfft);  A=0.05;
y = [A*(rand(1,Nd)-0.5+j*(rand(1,Nd)-0.5)) y];
y = [y A*(rand(1,Nd)-0.5+j*(rand(1,Nd)-0.5))];
if nF>0
  y=[A*(rand(1,nF)-0.5+j*(rand(1,nF)-0.5)) y(1:end-nF)]; %Delayed
 elseif nF<0
  y=[y(1-nF:end) A*(rand(1,-nF)-0.5+j*(rand(1,-nF)-0.5))]; %Advanced
end
% IFO (Integral Frequency Offset) estimation
IFO_range = [-3:3];
y_ifft = y(Nd+Ng+[1:Nfft]);
IFO_est = IFO_estimate(y_ifft,PRS,IFO_range); % Eq.(11.3.10a,b,c)
%In order to realize how critical the IFO estimate is for FFS,
% activate the following statement even with a very short delay nF=1;
% IFO_est = zeros(1,3);
% FFS (fine frame synchronization)
Y = fft(y_ifft,Nfft);
YX = Y.*conj(rotate_r(PRS,IFO_est));
[Max,nF_] = max(abs(ifft(YX)));  % Eq.(11.3.11)
if nF_>Nfft/2,  nF_=nF_-Nfft;  end % Periodicity of FFT/IFFT
nF_h = (nF_-1)
% FFO (Fractional Frequency Offset) estimation
nn = Nd+[1:Ng]; nn1 = Nfft + nn;
% To realize the importance of reflecting the FFS into FFO estimation,
% activate the following statement with a delay nF=2;
% nF_h = 0;
FFO_est = angle(y(nF_h+nn1)*y(nF_h+nn)')/(2*pi) % Eq.(11.3.12)
% FFO estimate without incorporating the FFS result
FFO_est0 = angle(y(nn1)*y(nn)')/(2*pi) % Eq.(11.3.12) without nF_h
% Overall CFO estimate
CFO_est = IFO_est + FFO_est;
fprintf('\n For CFO=%10.8f,\n', CFO);
form = '  CFO_estimate(%10.8f) = IFO(%10.8f)+FFO(%10.8f)\n';
for i=1:3
   fprintf(form, CFO_est(i), IFO_est(i), FFO_est);
end
% CFO compensated symbols
y_compensated = compensate_CFO(y,CFO_est(3),Nfft);
% Discrepancy between the CFO compensated symbols and original ones
discrepancy = norm(tx-y_compensated(Nd+[1:L_tx]))/L_tx
```

The above program "do_sync_for_DMB.m" simulates the synchronization scheme depicted in Fig. 11.10(a) where the routines 'IFO_estimate()' and 'phase_ref_symbol()' are used to compute the IFO estimates by Eqs. (11.3.10a~c) and to generate the frequency-domain PRS (phase reference symbol), respectively.

11.4 CHANNEL ESTIMATION AND EQUALIZATION

A frequency-domain training sequence $X(k) = X_R(k) + jX_I(k)$ with the corresponding channel output $Y(k) = Y_R(k) + jY_I(k)$ can be used for channel estimation as

$$\hat{H}(k) = \frac{Y(k)}{X(k)} = \frac{X^*(k)Y(k)}{X^*(k)X(k)} = \frac{\big(X_R(k)Y_R(k)+X_I(k)Y_I(k)\big)+j\big(X_R(k)Y_I(k)-X_I(k)Y_R(k)\big)}{X_R^2(k)+X_I^2(k)} \quad (11.4.1)$$

where $X(k)$ and $Y(k)$ are the FFTs of the (long preamble) input and the corresponding output of the channel, respectively, $X_R(k)$ and $Y_R(k)$ are the real parts, and $X_I(k)$ and $Y_I(k)$ are the imaginary parts of $X(k)$ and $Y(k)$, respectively. Noting that in the IEEE Standard 802.11a, the long preamble used for channel estimation is $X(k) = X_R(k) + jX_I(k) = 1$ or -1 (with $X_I(k) = 0$) as shown in Fig. 11.6(b), the channel estimator (11.4.1) can be simplified as

$$\hat{H}(k) = \frac{Y(k)}{X(k)} = X_R(k)\big(Y_R(k)+jY_I(k)\big) = X(k)Y(k) \quad (11.4.2)$$

Since the long preamble contains two repeated training sequences, the average of the FFTs ($Y_1(k)$ and $Y_2(k)$) of the channel outputs to the two consecutive training sequences can be taken for better channel estimation:

$$\hat{H}(k) = \frac{1}{2}X(k)\big(Y_1(k)+Y_2(k)\big) \quad (11.4.3)$$

This estimation scheme is cast into the MATLAB routine 'channel_estimate()'. The following program "do_channel_estimation.m" uses this routine to estimate a channel based on the frequency-domain long preamble $X(k)$ and the FFT $Y(k)$ of the output of the channel to the (time-domain) long preamble. It also uses another routine 'equalizer_in_freq()' to equalize the output to the unknown input by compensating the channel effect:

$$\hat{X}(k) = \frac{Y(k)}{\hat{H}(k)} \quad (11.4.4)$$

```
function H_est=channel_estimate(X,y)
% X = Known frequency-domain training symbol
% y = Time-domain output of the channel
% H_est = Estimate of the channel (frequency) response
if length(X)>52,  X = X([1:26 28:53]);  end   % for k=[-26:-1  1:26]
y1 = y(32+[1:64]);  y2 = y(96+[1:64]);
Y1 = fft(y1); Y1 = Y1([39:64 2:27]); % Arranged in the -/+ frequency
Y2 = fft(y2); Y2 = Y2([39:64 2:27]); % Arranged in the -/+ frequency
H_est = X.*(Y1+Y2)/2; % Eq.(11.4.3)
```

```
%do_channel_estimation.m
clear, clf
% Read the time-domain channel response stored in a 64x2 matrix
load  ch_complex.dat;
h=(ch_complex(:,1)+j*ch_complex(:,2)).'; % Time-domain channel response
Nfft=64; Ng=16; Nsym=Nfft+Ng; Nnull=Nsym; Nw=2*Ng; Tsym=4e-6;
A_sig=0.5; A_noise=0.01; % Amplitudes of signal and noise
Nd=120; % Remaining period of the last symbol in the previous frame
[s_preamble,Short] = short_train_seq(Nfft);
[l_preamble,Long] = long_train_seq(Nfft);
% Make a pseudo received sequence
t_frame=[A_sig*(rand(1,Nd)-0.5) zeros(1,Nnull) s_preamble l_preamble];
symbol=A_sig*(rand(1,Nd)-0.5); symbol=[symbol(end-Ng+1:end) symbol];
t_frame = [t_frame symbol];  L_frame = length(t_frame);
noise = A_noise*(rand(1,L_frame)-0.5 +j*(rand(1,L_frame)-0.5));
r_frame = channel(t_frame,h) + noise;
True_STO = Nd+Nnull+Nsym*2+1 % Starting point of the long preamble
STO = True_STO
% STO estimation is critical to the performance of channel estimation
% To realize this, change the above line into STO=True_STO+1 or -1
H_est = channel_estimate(Long,r_frame(STO+[0:159]));
% The true frequency-domain channel response is obtained from the FFT
%  of the time-domain channel (impulse) response.
H = fft(h,Nfft); % k=0(1):26(27)  27(28):37(38)  38(39):63(64)
H_true = H([39:64 2:27]); % Arranged in -/+ frequency k=-26:-1  1:26
discrepancy_H_est_and_H_true = norm(H_est-H_true)/norm(H_true)
% Let's see how the channel equalizer with the estimated channel
%  response (H_est) works.
X = rand(1,52)-0.5+j*(rand(1,52)-0.5); % for k=[-26:-1  1:26]
X_arranged = [0 X(27:end) zeros(1,11) X(1:26)]; % in +/- frequency
x = ifft(X_arranged); x_CP = [x(49:64) x];  % IFFT and add CP
y = channel(x_CP,h); % Channel output to an arbitrary input with CP
Y = fft(y(17:80)); % Remove CP and FFT
Y=Y([39:64 2:27]); % Arranged in -/+ frequency for k=[-26:-1 1:26]
Yeq = equalizer_in_freq(Y,H_est);
discrepancy_X_and_Y = norm(X-Y)/norm(X) % With no channel compensation
discrepancy_X_and_Yeq = norm(X-Yeq)/norm(X) % With channel equalizer
```

```
function [y,ch_buf] = channel(x,h,ch_buf)
L_x = length(x); L_h = length(h); h=h(:);
if nargin<3, ch_buf = zeros(1,L_h);
 else L_ch_buf = length(ch_buf);
      if L_ch_buf<L_h,  ch_buf = [ch_buf zeros(1,L_h-L_h)];
       else  ch_buf = ch_buf(1:L_h);
      end
end
for n=1:L_x,  ch_buf=[x(n) ch_buf(1:end-1)];  y(n)=ch_buf*h;  end
```

```
function Y_eq = equalizer_in_freq(Y,H_est)
H_est(find(abs(H_est)<1e-6))=1;  Y_eq = Y./H_est;  % Eq.(11.4.4)
```

```
% ch_complex.dat
 0.492366762830447834 9754    -0.372182474243322847 2294
-0.517511464802862475 3096    -0.304331371830144081 7804
...................
```

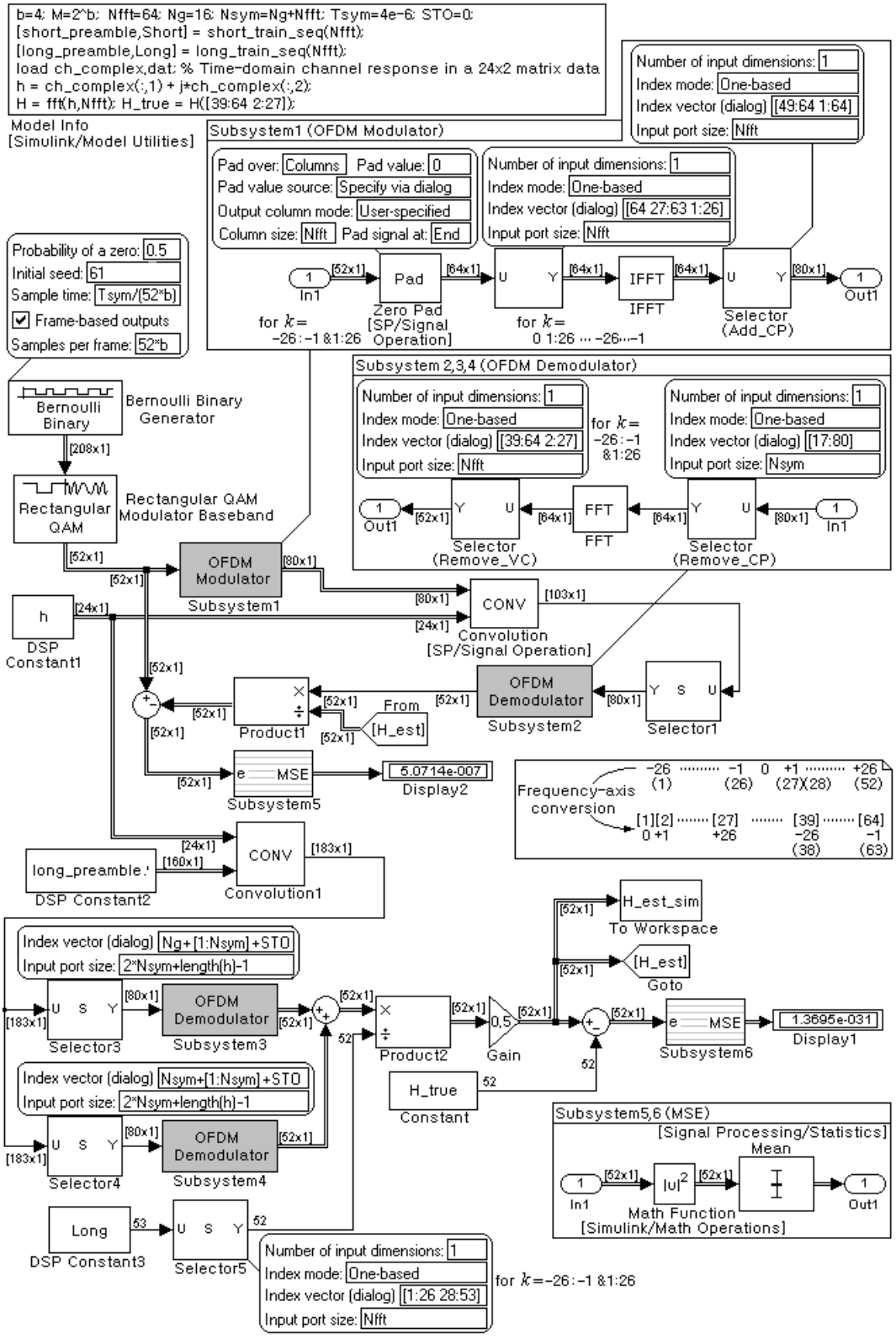


Figure 11.13 Simulink model for channel estimation and compensation ("channel_estimation_sim.mdl")

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